Mathematics 3820H – Mathematics from medieval to modern times

TRENT UNIVERSITY, Fall 2020

Take-home Final Examination

Due just before midnight on Friday, 18 December.

Instructions: Give complete answers to receive full credit, including references to any and all sources you used. You may ask the instructor to clarify the instructions or any of the questions, use a calculator or computer to perform any necessary calculations, and consult any sources you wish, *with the exception of other students' work,* and *you may not give or receive any other aid on this exam, except with the instructor's explicit permission.* If submission via Blackboard fails, please email your solutions to the instructor at: `sbilaniuk@trentu.ca`

Part • – Potpourri. Do *all three* (3) of **1** – **3**.

1. Answer all of **a** – **j**. *[10 = 10 × 1 each]*

- **a.** Name a mathematician who is better known nowadays as a poet.
- **b.** Name a mathematician who was reputed to be a sorcerer.
- **c.** Name an early critic of the lack of rigour in the foundations of calculus.
- **d.** Who first used the symbol + to denote addition?
- **e.** Name three mathematicians who were also astrologers.
- **f.** What were complex numbers invented for?
- **g.** Who established the complex numbers as a number system in its own right?
- **h.** Name two mathematicians who worked as code breakers for their governments.
- **i.** Name three mathematicians who were also civil servants, in positions unrelated to their mathematical careers.
- **j.** Who was the most prolific (in terms of page count) mathematician ever?
- **k.** Which mathematician has the greatest number of distinct publications? *[0.5 bonus]*

2. After calculus was introduced by Newton and Leibniz, mathematicians and scientists used and continued to develop it for nearly two centuries before it was given a more or less rigourous foundation. Explain why they did so and try to assess whether they were justified in doing so. *[13]*

3. Recall from lecture that a natural number n is *congruous* if there are natural numbers a and b with $a > b$ such that $n = ab(a-b)(a+b)$ (if $a+b$ is even) or $n = 4ab(a-b)(a+b)$ (if $a+b$ is odd). Prove Fibonacci's result that if n is congruous, then it is divisible by 24. *[12]*

Hint: $24 = 3 \cdot 8$, modular arithmetic, and a little brute force go a long way here.

[**Parts •• – •••• are on page 2.**]

[**Part •** is on page 1.]

Part •• – Questions. Do *one* (1) of **4** or **5**.

4. Was the development of a place-value system for writing numbers necessary for the development of modern algebra and calculus? Explain! *[15]*

5. Give a brief sketch of the life and career of Giovanni Saccheri, as well as the principal results he is known for nowadays. *[15]*

Part ••• – Reasoning. Do any *two* (2) of **6** – **8**.

6. State Āryabhaṭa's formula for the volume of a triangular pyramid (p. 26 of Clark's translation of the *Āryabhaṭīya*) in modern terms and then prove it or show that it is incorrect. *[10]*

You can find this translation of the *Āryabhaṭīya* on Blackboard, on the course web page, or at Wilbour Hall/Namas Te:

`https://www.wilbourhall.org/pdfs/aryabhatiyaEnglish.pdf`

7. State and prove Newton's Generalized Binomial Theorem. *[10]*

8. We saw that a particular ellipse could be generated by an epicycle in the lecture on Kepler. Show that every ellipse can be generated by some epicycle. *[10]*

[Total = 70]

Part •••• – Simplicity? Bonus problems!

1! + 2! + 3!. Write an original poem touching on mathematics or its history. *[1]*

1 + 2 + 3 + 4. The Romantic poet Samuel Taylor Coleridge, best known for works such as *Kubla Khan* and *The Rime of the Ancient Mariner,* once wrote a poem describing Euclid's Proposition I-1 and its proof. Find another example of a poem giving the statement of a mathematical theorem and its proof. *[1]*

I hope that you enjoyed the course
in spite of all the glitches.
Have a good break!