Leonhard Euler (1707-1783)

- pronounced "oiler" [not a hockey player]
- born and raised in Basel in Switzerland; his father had taken courses from Jacob Bernoulli but became a minister
- enrolled in the Univ. of Basel at 13 and studied with Johann Bernoulli
  - got an M.A. in 1723, thesis comparing Newton to Descartes
  - a Ph.D. in 1726 with a thesis on the propagation of sound.
  - then obtained a position at the Academy of Sciences in St. Petersburg in 1727 (partly on the recommendation of his friend & collaborator Daniel Bernoulli)
  - moved on to the Academy of Sciences in Berlin in 1741, and then back to St. Peterburg in 1766 (until he died)

Probably the most prolific mathematician ever:

- ≥ 800 publications
- collected works fill 74 volumes of publications
  - several more each of letters and of notebooks & unpublished material.
- One of the very really top flight mathematicians who wrote textbooks
  - a text on algebra
  - on calculus
  - his most popular publication in his lifetime was a series of letters explaining various mathematical & scientific topics to a German princess he tutored.
- he did this inspite serious health problems
  - went blind in one eye in 1738
  - blind in the other in 1766 after failed surgery on a cataract in that eye.
  - his productivity actually increased, if anything, since other people wrotedown what he dictated.

Worked in many areas of math & science, an incomplete list follows:

- astronomy: computational techniques for computing orbits
- geography: - methods for calculating longitude precisely
  - techniques for drawing maps
- optics: preferred Huygen's wave theory of light to Newton's particle theory of light.
- statics: eg equations for handling forces acting on a beam
- fluid dynamics: ship design
- graph theory: Königsberg bridge problem [look it up!]
- power series expansions and was one of the early users of these expansions for e^x & $\ln(x)$ in particular
- number theory: eg - showed that $\sum_{p \text{ prime}} \frac{1}{p}$ actually diverges
  - showed that every even perfect number has the form $2^{p-1}(2^p - 1)$ for $p, 2^p - 1$ prime. [The converse was known to Euclid.]
  - conjectured the law of quadratic reciprocity but couldn't prove it.

- introduced the totient function.

$\varphi(n)$ = # positive integers $\leq n$ which are relatively prime to n

(If p is prime, $\varphi(p) = p-1$.)

& proved <u>Euler's Theorem</u>: (1763)

If $m \geq 1$ and is relatively prime to a, then $a^{\varphi(m)} \equiv 1 \pmod{m}$.

[This is a generalization of Fermat's Little Thm., which was first proved by Euler in 1736.]

- Euler's Formula: $e^{i\varphi} = \cos(\varphi) + i\sin(\varphi)$ (1740, published in 1748)

[Precursors to this were known to Abraham de Moivre (1667-1754)

eg 1722: $(\cos(x) + i\sin(x))^n = \cos(nx) + i\sin(nx)$ (n integer)

]

Why does Euler's formula work?

Like Euler, we'll use the power series expansion of $e^x = 1 + x + \frac{x^2}{2} + \frac{x^3}{6} + \cdots + \frac{x^n}{n!} + \cdots = \sum_{n=0}^{\infty} \frac{x^n}{n!}$.

$$e^{i\varphi} = 1 + i\varphi + \frac{i^2\varphi^2}{2} + \frac{i^3\varphi^3}{6} + \cdots + \frac{i^n\varphi^n}{n!} + \cdots \qquad \text{but } i^2 = -1$$

$$= 1 + i\varphi - \frac{\varphi^2}{2} - i\frac{\varphi^3}{6} + \frac{\varphi^4}{24} + \cdots$$

$$= \left(1 - \frac{\varphi^2}{2} + \frac{\varphi^4}{24} - \cdots + \frac{(-1)^k \varphi^{2k}}{(2k)!} + \cdots\right)$$

$$+ i\left(\varphi - \frac{\varphi^3}{6} + \cdots + \frac{(-1)^k \varphi^{2k+1}}{(2k+1)!} + \cdots\right)$$

$$= \cos(\varphi) + i\sin(\varphi).$$

Read: Leonhard Euler: His Life, the Man, and his Works

~~http~~ www.cs.purdue.edu/homes/wxg/EulerLect.pdf.