The Isaacs - Barrow and Newton

Isaac Barrow (1630-1677)

- mathematician and theologian
- he worked on geometry and optics, and calculus
  - in particular he worked on finding tangents to algebraic curves (Complements Wallis' work on integrating algebraic curves.)

ex To find the slope of $y = x^2$ at x

he would add a very small number o to x

and see how y changed:

$$\Delta y = (x+o)^2 - x^2$$

$$= x^2 + 2ox + o^2 - x^2 = 2xo + o^2$$

Since o is very small (infinitesimal...), o^2 is even smaller and so can be neglected,

so a change of o in x changes y by $2xo$.

$\therefore$ the slope of $y = x^2$ at x is $2x$.

Barrow helped discover the Fundamental Theorem of Calculus.

He taught Newton, and Newton helped him prepare lectures and at least one book on geometry for publication. Barrow resigned the Lucasian Professorship in Mathematics at Cambridge in favour of Newton in 1669, and then spent the rest of his life focussing on theology.

Lucasian Professor of Mathematics

- post was established in 1663 with Barrow as the first. (received a royal charter in 1664)
- Newton was the second holder
- held by various prominent mathematicians & physicists since then (includes Stokes, Babbage, Dirac, Hawking, and (according to Star Trek) will be held by the android Data in the late 2300s)

Isaac Newton (1642-1727)

- born into a family of "landed gentry"
- was an undergraduate at Cambridge & studied under Barrow
- ended up as a teaching and research assistant to Barrow while still an undergraduate, helping to prepare lectures and at least one book
- In 1665-1666 England had an outbreak of the bubonic plague, resulting in Oxford and Cambridge shutting down for a while. Newton went home and thought about math & science. By his own account (later on) he discovered his law of gravitation and developed his first cut at differential calculus in this period.

He did not immediately publish, because he wanted to refine and extend his results.

eg He showed the law of gravitation resulted in Kepler's Laws of planetary motion, but he wanted to show this was true if the planets were spheres instead of just points. (It took him until the 1680s to sort that out.)

Somewhen as an undergraduate he also developed

Newton's Binomial Theorem (for integer & fractional exponents)

$$(1+x)^a = 1 + ax + \frac{a(a-1)}{2}x^2 + \frac{a(a-1)(a-2)}{6}x^3 + \cdots + \frac{a(a-1)\cdots(a-n+1)}{n!}x^n + \cdots$$

This allowed him to expand many functions as infinite series and apply Barrow's method of tangents & Wallis' methods of computing integrals of powers of x to find general tangents and areas under curves.

eg Wallis couldn't integrate $\int_0^{\pi} \sqrt{1-x^2}\,dx$

(although trying to gave him the Wallis product)

but Newton could expand $\sqrt{1-x^2} = (1-x^2)^{1/2}$

& then integrate term-by-term using Wallis' formulas (power rule of integration).

This gave Newton his first cut at differential & later integral) calculus.

His major work in physics is

Philosophiae Naturalis Principia Mathematica

of which the first edition appeared in 1686.
He had developed a lot of the results using calculus, but for publication he replaced all the arguments by arguments based on Euclidean geometry. (This made it easier for many of his contemporaries to understand, but harder for us.)

He didn't actually publish his work on differential calculus until 1692, and on integral calculus until 1704.
Earlier, in the ~~1670s~~ 1660s & 1670s he circulated manuscripts calculus among some other mathematicians and scientists in England.

He called derivatives "fluxions" and wrote them as $\dot{x}, \ddot{x}, \ldots$ where every derivative is implicitly a derivative with respect to time. ie $\dot{x} = \frac{dx}{dt}$. Following Barrow he would write o for dt and $\dot{x}o$ for $dx \left(= \frac{dx}{dt} dt\right)$.

Integrals ("fluents") he wrote as $\dot{x}$.

In 1696 he became Warden of the Mint (nominal boss)
in 1699 —. " — Master of the Mint (actual boss)
and then resigned from the Lucasian Professorship and Cambridge in 1701. He worked as the Master of the Mint until his death. (His duties included overseeing the production of money - including testing the purity of the alloys, developing machines to improve the consistency and shape of the coins, etc. - as well as hunting counterfeiters, some of which he did personally.)

- in his private life he also worked on alchemy and theology.

Very important as a physicist: - laws of motion
- universal law of gravitation
- optics - white light is a mix of colours which refract differently
- invented reflecting telescope designs (and methods for shaping the mirrors)