The French Connection II

René Descartes (1596-1650)

- was a soldier as a young man (mainly for the Dutch)
- worked on philosophy, physics, as well as mathematics

Philosophy: used skepticism and rationality as a method

Physics: Did work in optics and an early universal theory of physics (theory of vortices).

Mathematics: Published a work called La Géométre as an appendix to his work Discours de la méthode (along with appendices on optics and on meteorology).

In it he presented the Cartesian coordinate system with applications to geometry. (Founded "analytic geometry".)
Adopted the idea of using x, y, z as variables and $a, b, c, \ldots$ as constants. Also used superscripts to represent powers, eg x^3.

- he had few (and not always complete) proofs

For example he stated the following theorem without proof:

Descartes' Rule of Signs

Suppose $p(x) = a_n x^n + a_{n-1} x^{n-1} + \cdots + a_1 x + a_0$
is a polynomial with real coefficients.
Then the number of positive roots of $p(x)$
is either equal to the number of sign changes
in the sequence $a_n, a_{n-1}, \cdots, a_1, a_0$ [omitting $a_k = 0$]
or it differs from this number by an even number.

Eg $x^3 - x$ sequence: $1, \cancel{0}, -1, \cancel{0}$ one sign change.
$\|$ & hence 1 (or 0) positive roots
$x(x-1)(x+1)$
one positive root

Pierre de Fermat (1601-1665)

- made his living as a lawyer [councillor and magistrate (for the local parliament)].
- didn't publish most of his work; we know most of it from his correspondence with Mersenne, Pascal, and others, as well as notes found after his death.
- invented Cartesian coordinates and analytic geometry at about the same time Descartes did
  - used it to study tangents, maxima & minima, and areas under curves
- collaborated with Pascal by correspondence on probability theory.
- best known for his work in number theory.

  eg 1) Every odd prime can be written as a difference of squares in exactly one way.

  2) Every prime of the form $4n+1$ can be written as a sum of two squares in exactly one way

(This complements a previous result of Diophantus that every prime of the form $4n-1$ cannot be written as a sum of two squares.)

3) Fermat's Little Theorem

If p is prime and $p \nmid a$, then $p \mid (a^{p-1}-1)$.

4) Fermat's Last Theorem

$x^n + y^n = z^n$ has no solutions in positive integers x, y, z for $n \geq 3$.

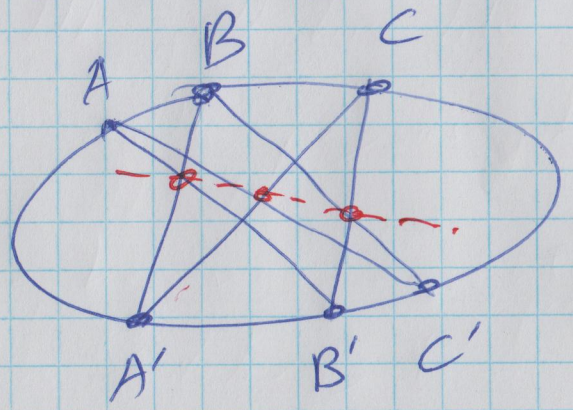
The statement occurs in a note in the margin of his copy of Diophantus' Arithmetica, but gives no proof (because the margin was too small). He did give a proof elsewhere for the case $n = 4$.
(No idea what general proof he thought he had; the general result was finally proved by Andrew Wiles in 1994.)

Blaise Pascal (1623-1662)

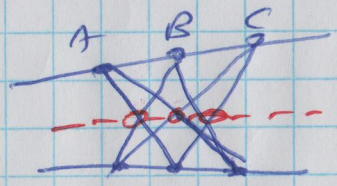
- child prodigy (educated by his father), suffered from poor health all his life
- worked in math, physics, philosophy, was also an inventor & a writer
- as a writer he was one of the modern pioneers of essays as a literary form
- as an inventor he is best known for creating a mechanical adding device
- as a physicist he mainly worked on hydrodynamics & hydrostatics, (has an unit of pressure named after him)
- as a philosopher tried to reconcile faith with rationalism.
- as a philosopher of science he anticipated the notion of falsifiability for theories

- as a mathematician he worked on geometry

eg Pascal's Hexagon Theorem (related to Pappus' Theorem



⊂ conic curve



)

(This gets mentioned in his Essay on Conics.)

- on algebra: Pascal's Identity

$$(n+1)^{k+1} - 1 = \sum_{m=1}^{n} \left((m+1)^{k+1} - m^{k+1} \right)$$

$$= \sum_{p=0}^{k} \binom{k+1}{p} \left(1^p + 2^p + 3^p + \cdots + n^p \right)$$

- on probability theory with Fermat by correspondence
  - devised the concept of expected value

Pascal's Wager (an argument for believing in God)

If there is no God and you believe (or not), it makes no difference.

If there is a God and you believe (and behave!), you get an infinitely good payoff. (eternity in heaven)

If there is a God and you don't believe (or don't behave), you get an infinetely bad payoff. (eternity in hell).