2020-10-21

Bonaventura Cavalieri (1598-1647)

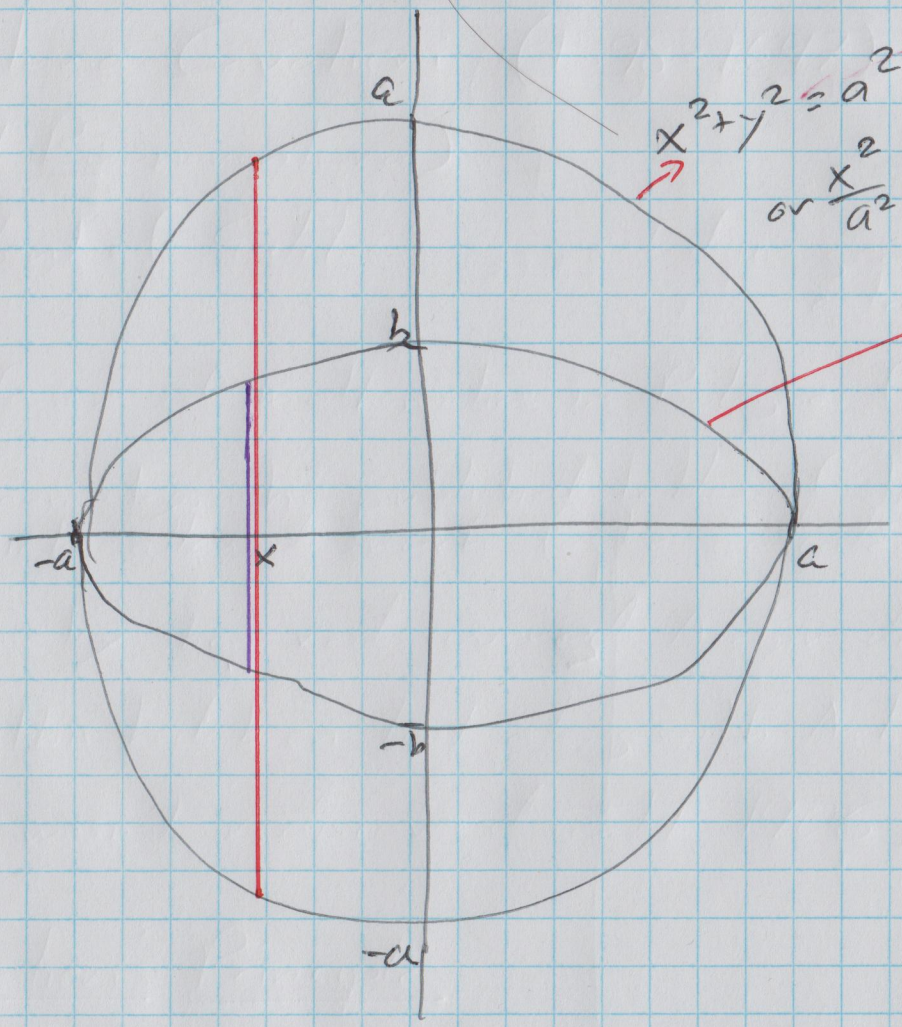
(also spelled Buonaventura)

- student of Galileo's, worked in geometry, trigonometry, astronomy, & optics.
- professor of mathematics in Bologna from 1629 on.
- in trigonometry, he published tables of trig values and their logarithms which allowed much faster calculations
- in geometry he rediscovered Archimedes' method of indivisibles (this had been lost along with Archimedes' Method, which wasn't rediscovered until 1906)

Cavalieri's Principle

If two objects have corresponding cross-sections that are always in the same ratio in length (resp. area) then the objects have their area (resp. volume) in that ratio, too.

eg



$x^2+y^2=a^2$ This has area πa^2.

or $\frac{x^2}{a^2}+\frac{y^2}{a^2}=1$

$\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$

$b<a$

What is the area?

The cross-section of the ellipse at x, is $2b\sqrt{1-\frac{x^2}{a^2}}$.

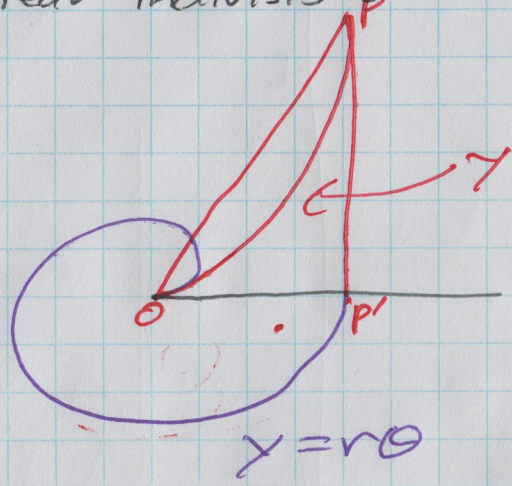
Similarly the cross-section of the circle at x is $2a\sqrt{1-\frac{x^2}{a^2}}$

The ratio of cross-sections at x is $\frac{2b\sqrt{1-\frac{x^2}{a^2}}}{2a\sqrt{1-\frac{x^2}{a^2}}} = \frac{b}{a}$.

$\therefore$ Area of the ellipse $= \frac{b}{a}\cdot\pi a^2 = \pi ab$.

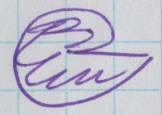
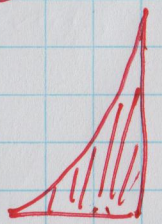
He used these ideas in very sophisticated ways.

eg "curvilinear indivisibles"



As long as $|PP'|$
$= 2\pi |OP'|$,

then

area  = area 

Cavalieri's Quadrature Formula

$$\int_0^a x^n dx = \frac{a^{n+1}}{n+1}$$

(for $n \geq 0$)

[He had a geometric statement equivalent to this.]

Others extended this formula to negative n and then to rational exponents in the later 1600's.

His work spawned a lot of effort in "infinitesimal analysis" that anticipated parts of calculus.

Last Lecture before Reading Week. See you after!