The Solution of the Cubic & Quartic Equations

Mainly due to four mathematicians:

Scipione del Ferro (c 1465 – 1526) [solved cubics of the form $x^3 + px = q$, where $p, q > 0$]

Niccolo Tartaglia (c. 1500 - 1557) [solved cubics of the form $x^3 + px = q$ & $x^3 + px^2 = q$, where $p, q > 0$]

Girolamo Cardano (c 1501 - 1576) [general solution to the cubic]

Ludovico Ferrari (1522 - 1565) [general solution to the quartic equation]

Scipione del Ferro – professor of mathematics at the University of Bologna [oldest university in Europe, founded in 1088]

- he passed on his solution to $x^3 + px = q$ $(p, q > 0)$ to his student Antonio Fiore
- he didn't publish, probably because reputation was built by solving challenge problems from others so having secret methods was an asset.

del Ferro's / Tartaglia's solution to $x^3+px=q$ $(p,q>0)$ (2)

(in modern style)

Let $x=u-v$, where $uv=\frac{p}{3}$.

Then $(u-v)^3+p(u-v)=q$

$$=u^3-3u^2v+3uv^2-v^3+pu-pv \qquad \text{but } v=\frac{p}{3u}$$

$$=u^3-3u^2\left(\frac{p}{3u}\right)+3u\left(\frac{p}{3u}\right)^2-\left(\frac{p}{3u}\right)^3+pu-p\left(\frac{p}{3u}\right)$$

$$=u^3-\cancel{pu}+\cancel{\frac{p^2}{3u}}-\frac{p^3}{27u^3}+\cancel{pu}-\cancel{\frac{p^2}{3u}}$$

$$\therefore \quad u^3-\frac{p^3}{27u^3}=q \implies u^6-\frac{p^3}{27}=qu^3$$

$$\implies u^6-qu^3-\frac{p^3}{27}=0 \implies (u^3)^2-q(u^3)-\frac{p^3}{27}=0$$

This is a quadratic in u^3, so we can use the quadratic formula:

$$u^3=\frac{-(-q)\pm\sqrt{(-q)^2-4\cdot1\cdot\left(-\frac{p^3}{27}\right)}}{2\cdot1}$$

$$=\frac{q\pm\sqrt{q^2-\frac{4p^3}{27}}}{2}$$

Since $x = u - v = u - \frac{p}{3u}$,

where u is the (real) cube root of $\frac{q + \sqrt{q^2 + 4\frac{p^3}{27}}}{2}$,

we ~~the~~ have $\underline{\underline{a}}$ solution to $x^3 + px = q$. //

Niccolo Tartaglia – worked out $x^3 + px = q$ independently and also how to handle $x^3 + px^2 = q$ $(p, q > 0)$.

- grew up in poverty and was self-taught
- worked as a mathematician, military engineer, surveyor, and a bookkeeper
- translated Archimedes and Euclid into Italian (one consequence of printing was the rise of the vernacular and a slow decline of Latin as <u>the</u> language of scholarship)
- wrote a number of original works [at least one of which he plagiarized]
  - reinvented Pascal's Triangle
  - gave an expression for the volume of a tetrahedron in terms of the distances between the vertices [a sort of generalization of Heron's formula for the area of a triangle]

He revealed his solutions to cubic equations to Cardano on condition that Cardano swear an oath of secrecy, which Cardano apparently did.

Cardano later published these techniques as part of his general solution to the cubic, using del Ferro's work as the basis for what Tartaglia had revealed. Cardano gave due credit to del Ferro & Tartaglia, but Tartaglia was not appeased...

next time: Cardano
& Ferrari