Abu Amir Yusuf ibn Ahmad al-Mutaman ibn Hud

(?-1085) [Emir of Zaragoza 1081-1085]

Wrote one major work, Kitab al-Istikmal ("Book of Perfection") in which he tried to compile, summarize, and synthesize, and replace previous Greek and Islamic works in mathematics. Completed one a projected two volumes before becoming Emir. Sources included Euclid, Apollonius, Archimedes, ibn Qurra, and many others. He did not give attributions, but we can identify the sources by the content (and it at least one case by the mistakes he preserved - an Arabic translation of Menelaus' Sphaerica).

In a lot of cases he heavily modified the material he borrowed. (e.g. He did alternate computations for a lot of the results that can be found in Book XIII of Euclid's Elements.)

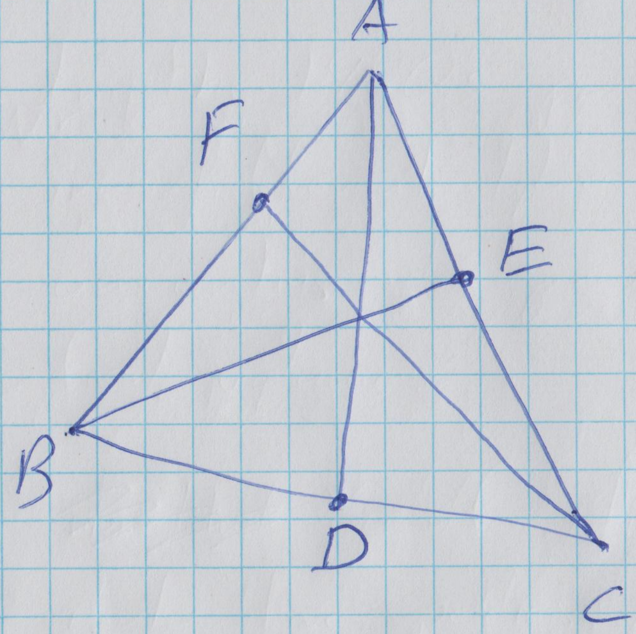
The work was lost for many centuries - all we had was a table of contents that were quoted by a later author.

In the 1980's parts of the Kitab al-Istikmal were discovered in different libraries [Copenhagen, Leiden, Damascus, Cairo], amounting to probably about 75% of the completed first volume.

~~Ibn Hud~~ Yusuf divided the "first genus" of mathematics into five species: 1. numbers & number theory
2. plane geometry
3. plane geometry (in combination with other topics)
4. solid geometry
5. solid geometry (in combination ...)

He also had original results.

One of the original results is <u>Ceva's Theorem</u> (rediscovered by Giovanni Ceva (1647-1734)).



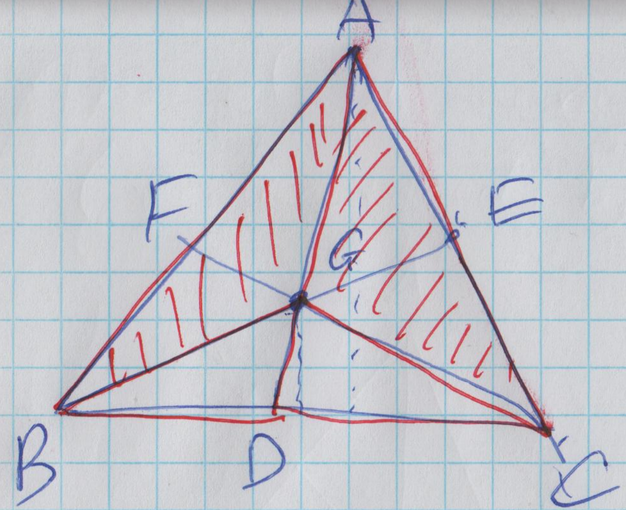
If D, E, F are points on sides BC, AC, AB (respectively) of $\triangle ABC$, then ~~BE~~, ~~AC~~, AD, BE, CF and ~~AB~~ meet in a point G if and only if

$$\frac{|AF|}{|FB|}\cdot\frac{|BD|}{|DC|}\cdot\frac{|CE|}{|EA|}=1.$$

Here's a proof, probably different from Yusuf al-Mu'taman's.

$\boxed{\Rightarrow}$ Suppose AD, BE, and CF all meet at a point G.

We need to show . We'll relate the lengths of AF, FB, <u>etc</u> to the areas of subtriangles in the diagram.



Let $|\Delta XYZ|$ = area of ΔXYZ,

ΔABD & ΔACD have the same height

$|\Delta ABD| = \frac{1}{2}$ base·height $= \frac{1}{2}|BD| \cdot \text{height}$

$|\Delta ACD| = \frac{1}{2}$ base·height $= \frac{1}{2}|CD| \cdot \text{height}$

So $\dfrac{|\Delta ABD|}{|\Delta ACD|} = \dfrac{|BD|}{|CD|}$.

Similarly, ΔGBD & ΔGCD have the same height

so $\dfrac{|\Delta GBD|}{|\Delta GCD|} = \dfrac{|BD|}{|CD|}$. It follows that

$$\frac{|\Delta AGB|}{|\Delta AGC|} = \frac{|\Delta ABD| - |\Delta GBD|}{|\Delta ACD| - |\Delta GCD|} = \frac{|BD|}{|CD|}.$$

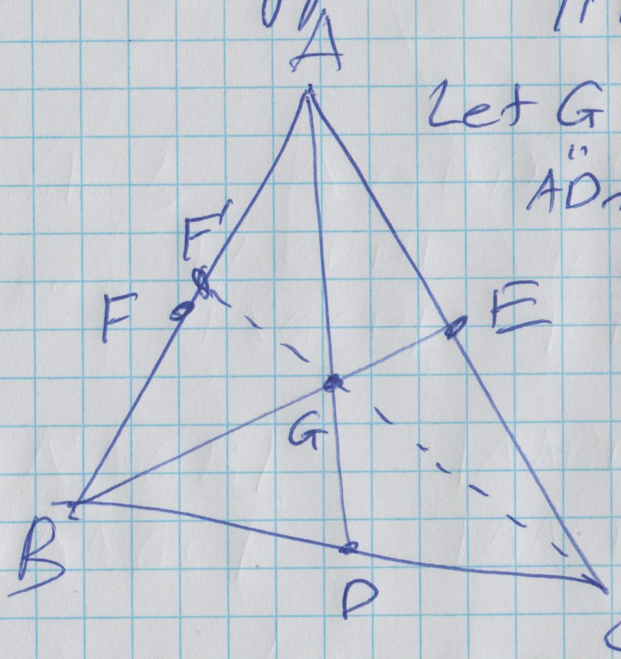
In a similar way, we can get $\dfrac{|\Delta AGB|}{|\Delta CGB|} = \dfrac{|AE|}{|CE|}$

& $\dfrac{|\Delta AGC|}{|\Delta CGB|} = \dfrac{|AF|}{|BF|}$.

Then $$\frac{|AF|}{|FB|}\cdot\frac{|BD|}{|DC|}\cdot\frac{|CE|}{|EA|} = \frac{\cancel{|\triangle AGC|}}{\cancel{|\triangle CGB|}}\cdot\frac{\cancel{|\triangle AGB|}}{\cancel{|\triangle AGC|}}\cdot\frac{\cancel{|\triangle CGB|}}{\cancel{|\triangle AGB|}}$$

$$= 1,$$ as desired.

$\boxed{\Leftarrow}$ Suppose $\dfrac{|AF|}{|FB|}\cdot\dfrac{|BD|}{|DC|}\cdot\dfrac{|CE|}{|EA|} = 1.$



Let $G \stackrel{..}{=} AD \cap BE$. Extend CG to AB at some point F'.

We'll show that $F = F'$.

By the $\Rightarrow$, we have

$$\frac{|AF'|}{|F'B|}\cdot\frac{|BD|}{|DC|}\cdot\frac{|CE|}{|EA|} = 1.$$

$$\therefore \frac{|AF'|}{|F'B|} = \frac{|AF|}{|FB|}$$

$$\Rightarrow F = F'$$

& $\therefore$ CF is also passing through G.

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