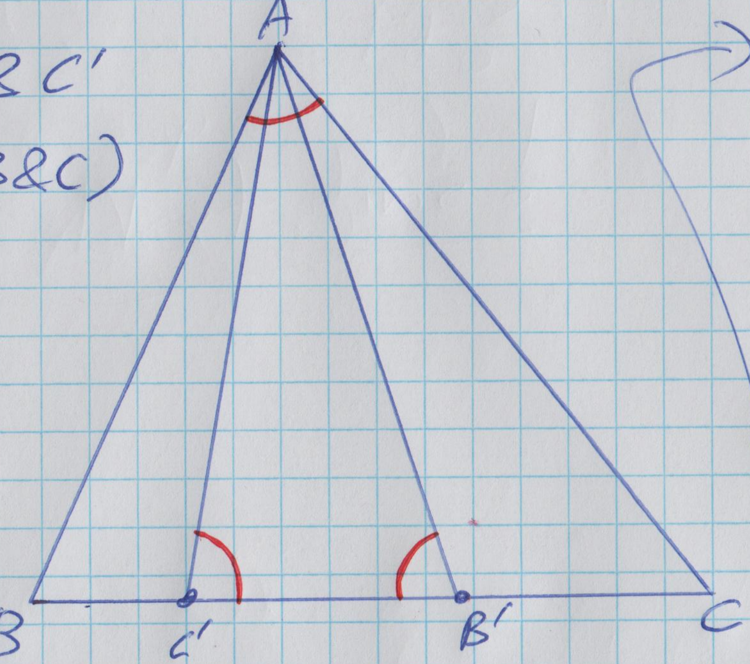
Some of Thabit ibn Qurra's work in geometry.

An extension of the Pythagorean Theorem to arbitrary triangles (different from the one given by Pappus).

Given a ΔABC, such that BC is the longest side. (i.e. $\angle BAC$ is the largest angle.)

Choose points B' & C' on BC (between B&C) so that

$\angle BAC$
$= \angle BB'A$
$= \angle AC'C$



Sides of similar are in the same proportion, so for some $p, q > 0$ we have

$|AB| = p|B'B| = q|C'A|$,

$|AC| = p|B'A| = q|C'C|$,

& $|BC| = p|BA| = q|AC|$

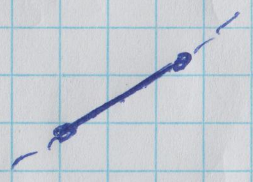
$\therefore |AB|^2 = \left(\frac{1}{p}|BC|\right)\cdot\left(p|BB'|\right)$
$= |BC|\cdot|BB'|$

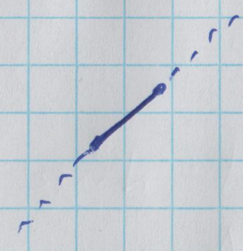
& $|AC|^2 = \left(\frac{1}{q}|BC|\right)\cdot\left(q|C'C|\right)$
$= |BC|\cdot|C'C|$, so

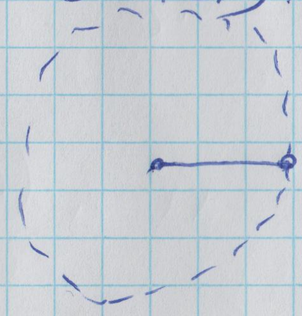
$|AB|^2 + |AC|^2 = |BC|\cdot|BB'| + |BC|\cdot|C'C|$
$= |BC|\cdot\left(|BB'| + |C'C|\right)$.

Then $\Delta B'BA$ and ΔBAC have two angles in common, so they are similar. Similarly, $\Delta AC'C$ is similar to ΔBAC.

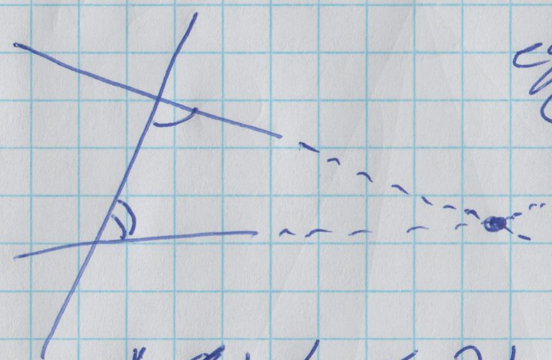
<u>Background:</u> Euclid gave the famous five Postulates as a foundation for plane geometry in his <u>Elements</u> ($\approx$ 300 B.C.).

I. 

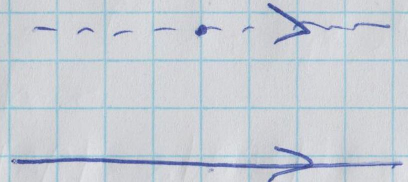
II. 

III 

IV $\llcorner = \llcorner$

V ("Parallel Postulate") 

$\angle + \angle < 2 \llcorner$

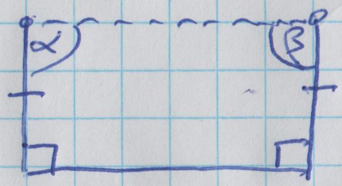
equivalent $\Longleftrightarrow$ V' "Playfair's Postulate" 

<u>Q.</u>: Do Postulates I–IV imply Postulate V ?
(or equivalent)

Many attempts were made and various people got various equivalents of Postulate V...

Thabit ibn Qurra wrote the effectively standard translation of the <u>Elements</u> into Arabic.

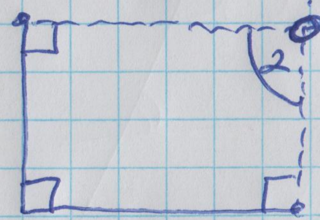
He proceeded to work on the Parallel Postulate

1° 

Postulates I–IV $\Rightarrow \angle\alpha = \angle\beta$

& $\angle\alpha = \angle\beta = \llcorner \Leftrightarrow$ Postulate V.

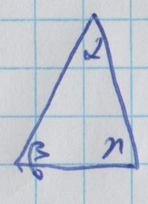
("Saccheri quadrilateral")

2° 

If $\angle\alpha = \llcorner$, however you do this, then Postulate V holds, & vice versa

("Lambert quadrilateral")

3° He also had results for triangles



$\alpha + \beta + \gamma = \llcorner + \llcorner \Leftrightarrow$ Postulate V

We'll revisit this with Saccheri & later workers on geometry