Mathematics 3820H – Mathematics from medieval to modern times
Trent University, Fall 2016

Take-home Final Examination
Due on Friday, 16 December, 2016.

Instructions: Give complete answers to receive full credit, including references to any and all sources you used. You may ask the instructor to clarify the instructions or any of the questions, use a calculator or computer to perform any necessary calculations, and consult any sources you wish, *with the exception of other students' work,* and *you may not give or receive any other aid on this exam, except with the instructor's explicit permission.*

Part I – This and that, ...   Do *all three* of **1 – 3**.

1. Answer all of **a – j**. *[10 = 10 × 1 each]*

   a. What is the origin of the word "algorithm"?

   b. Name three mathematicians who were also civil servants, in positions unrelated to their careers as mathematicians.

   c. Which mathematician accumulated the greatest lifetime page count?

   d. Name three mathematicians who were also inventors.

   e. Who was the first mathematician to draw the graph of a function?

   f. Name the previous holder of the professorship that Newton held at Cambridge.

   g. Who dedicated a book of mathematics in verse to his daughter?

   h. Who first used + to denote addition?

   i. Name three mathematicians who were also astrologers.

   j. What were complex numbers invented for?

2. David Hilbert gave a list of the then unsolved problems he thought were the most important to work on. Give a brief rundown of the list and its history, incuding which of the problems have been solved since and whether they are still considered significant. *[15]*

3. Read Chapter III (pp. 16-19) of Edward Strachey's translation of Bhaskara II's *Bija Ganita*, which you can read or download via Google Books at:

   `books.google.ca/books?id=StzuAAAAMAAJ`

   Explain the method of finding the square roots of algebraic expressions given in Section V (pp. 18-19) as fully as you can. *[10]*

Part II – ... and has-beens, ...   Do *one* (1) of **4** or **5**.

4. "The feature distinguishing mathematics from the natural sciences is that its ultimate validating principle is proof rather than experience." Do the invention and evolution of calculus support this assertion or not? *[15]*

5. Trace the spread and evolution of the Hindu number system and try to assess to what extent it spread because of its superiority to the available alternatives. *[15]*

   You may find *The Hindu-Arabic Numerals*, by L.C. Karpinski and D.E. Smith, available at Project Gutenberg (`www.gutenberg.org/ebooks/22599`), to be of interest.

   [Parts **III** and **IV** are on page 2.]

Part III − ... **and necessary things,** ...   Do any *two* (2) of **6** − **8**.

6. Give an informal description of how the use of infinitesimals in calculus can be made rigourous. *[10]*

7. Show that *Gregory's series*, $4 - \frac{4}{3} + \frac{4}{5} - \frac{4}{7} + \frac{4}{9} - \cdots$, converges to π and determine how many terms of it one must one add up to guarantee that the partial sum is within 0.01 of π? *[10]*

8. Suppose $n > 1$ is a natural number such that $p = 3 \cdot 2^{n-1} - 1$, $q = 3 \cdot 2^n - 1$, and $r = 9 \cdot 2^{2n-1} - 1$ are all prime numbers. Prove Thabit ibn Qurra's result that $a = p \cdot q \cdot 2^n$ and $b = r \cdot 2^n$ are a pair of amicable numbers (each is the sum of the other's divisors other than the other itself). *[10]*

[Total = 70]

Part IV - ... **and even verse things.**   Bonus!

$\boldsymbol{\phi}$. Write an original poem touching on mathematics or its history. *[1]*

$\boldsymbol{\pi}$. Find and give a complete reference to a poem touching on mathematics or its history that you did not write, and which your instructor has not seen before. *[1]*

I HOPE THAT YOU ENJOYED THE COURSE ...
AND YOUR BREAK IS EVEN MORE FUN!