Mathematics 3820H – Mathematics from medieval to modern times
Trent University, Fall 2014

Take-home Final Examination
Due on Wednesday, 17 December, 2014.

Instructions: Give complete answers to receive full credit, including references to any and all sources you used. You may ask the instructor to clarify the instructions or any of the questions, use a calculator or computer to perform any necessary calculations, and consult any sources you wish, *with the exception of other students' work,* and *you may not give or receive any other aid on this exam, except with the instructor's explicit permission.*

Part I – This and that, ... Do all three of **1** – **3**.

1. Answer all of **a** – **j**. *[10 = 10 × 1 each]*

- **a.** What is the origin of the word "algebra"?
- **b.** Name three mathematicians who were also civil servants, in positions unrelated to their careers as mathematicians.
- **c.** Name a mathematician who is better known nowadays as a poet?
- **d.** Who first denoted the ratio of a circle's circumference to its diameter by π?
- **e.** Who was the first mathematician to draw the graph of a function?
- **f.** Name three outstanding mathematicians who published much less than they might have due to being perfectionists.
- **g.** Who dedicated a book of mathematics in verse to his daughter?
- **h.** Who first used = to denote equality?
- **i.** Name three mathematicians who were also astrologers.
- **j.** Name the present holder of the professorship that Newton held at Cambridge.
- **k.** *Bonus!* Name a *future* holder of the professorship that Newton held. *[0.5]*

2. Trace the spread and evolution of the Hindu number system and try to assess to what extent it spread because of its superiority to the available alternatives. *[15]*

3. State and prove Brahmagupta's formula for the area of a cyclic quadrilateral. *[10]*

Part II – ... and has-beens, ... Do *one* of **4** or **5**.

4. After calculus was introduced by Newton and Leibniz, mathematicians and scientists used and continued to develop it for nearly two centuries before it was given a more or less rigourous foundation. Explain why they did so, and argue whether or not they were justified in doing so. *[15]*

5. To what extent was progress in mathematics before 1700 A.D. driven by progress in science and technology, and *vice versa*? *[15]*

[Parts **III** and **IV** on page 2.]

Part III – ... **and necessary things,** ... Do any *two* of **6** – **8**.

6. Prove that the nth Fibonacci number is the closest integer to $\frac{1}{\sqrt{5}}\left(\frac{1+\sqrt{5}}{2}\right)^n$. *[10]*

7. Suppose $f(x) = a_n x^n + a_{n-1}x^{n-1} + \cdots + a_1 x + a_0$ is a polynomial in x with integer coefficients. Can the values $f(0)$, $f(1)$, $f(2)$, ... all be prime numbers? If so, give an example of such a polynomial and show that it has this property; if not, prove that no polynomial in x with integer coefficients can have this property. *[10]*

Hint: Euler considered $p(x) = x^2 - x + 41$, which works for a while ...

8. Show that if p and $2^p - 1$ are both prime numbers, then $2^{p-1}(2^p - 1)$ is a perfect number. *[10]*

[Total = 70]

Part IV - ... **and even verse things.** Bonus!

α. Write an original poem touching on mathematics or its history. *[1]*

β. Find and give a complete reference to a poem touching on mathematics or its history that you did not write, and which your instructor has not seen before. *[1]*

I HOPE THAT YOU ENJOYED THE COURSE. HAVE A GOOD BREAK!