Mathematics 3820H – Mathematics from medieval to modern times TRENT UNIVERSITY, Fall 2012

TAKE-HOME FINAL EXAMINATION Due on Wednesday, 19 December, 2012.

Instructions: Give complete answers to receive full credit, including references to any and all sources you used. You may use your texts from this and any other courses, as well as any handouts, class notes, and the like; you may also ask the instructor to clarify the instructions or any of the questions; and you may use a calculator or computer to perform any necessary calculations. You may also consult any other sources you wish, *but you may not give or receive any other aid on this exam, except with the instructor's explicit permission.*

Part I – **This and that**, ... Do all three of 1 - 3.

- **1.** Answer all of $\mathbf{a} \mathbf{j}$. $[10 = 10 \times 1 \text{ each}]$
 - **a.** What is the origin of the word "algorithm"?
 - **b.** Name three mathematicians who were also civil servants, in positions unrelated to their careers as mathematicians.
 - c. Name three mathematicians who were also astronomers.
 - **d.** Which mathematician managed to collect letters of reference from both Newton and Leibniz but still couldn't get a professorship?
 - e. Who was the first mathematician to draw the graph of a function?
 - f. Who was the most prolific mathematician ever?
 - g. Who dedicated a book of mathematics in verse to his daughter?
 - **h.** Who tried to relate the orbits of the planets to the Platonic solids?
 - i. Name a mathematician who is better known nowadays as a poet?
 - **j.** Name the present holder of the professorship that Newton held at Cambridge.
 - **k.** Bonus! Name a future holder of the professorship that Newton held. [0.5]
- 2. "The feature distinguishing mathematics from the natural sciences is that its ultimate validating principle is proof rather than experience." Do the invention and evolution of calculus support this assertion or not? [15]
- **3.** Read Chapter III (pp. 16-19) of Edward Strachey's translation of Bhaskara II's *Bija Ganita*, which you can read or download via Google Books at:

http://books.google.ca/books?id=StzuAAAAMAAJ

Explain the method of finding the square roots of algebraic expressions given in Section V (pp. 18-19) as fully as you can. /10/

[Parts II - IV on page 2.]

Part II $- \ldots$ some has-beens and might-have-beens, \ldots Do one of 4 or 5.

- 4. To what extent was the development of mathematics after ancient times driven by the needs of astronomy? [15]
- 5. Trace the spread and evolution of the Hindu number system and try to assess to what extent it spread because of its superiority to the available alternatives. [15]

You may find *The Hindu-Arabic Numerals*, by Louis Charles Karpinski and David Eugene Smith, available at Project Gutenberg (www.gutenberg.org/ebooks/22599), to be of interest.

Part III $- \ldots$ and some necessary things. Do any *two* of 6 - 8.

- 6. State and prove Brahmagupta's formula for the area of a cyclic quadrilateral. [10]
- 7. Use the Dedekind cut or *schnitt* definition of real numbers to show both **a** and **b**.
 - **a.** The rational numbers are *dense* in the real numbers, *i.e.* given any real numbers s and t such that s < t, there is a rational number q such that s < q < t. [5]
 - **b.** Any set of real numbers with an upper bound has a least upper bound. [5]

8. Prove that the *n*th Fibonacci number is the closest integer to $\frac{1}{\sqrt{5}} \left(\frac{1+\sqrt{5}}{2}\right)^n$. [10]

|Total = 70|

Part IV - From bad to verse? Bonus!

 \bigcirc The Romantic poet Samuel Taylor Coleridge, best known for works such as *Kubla Khan* and *The Rime of the Ancient Mariner*, once wrote a poem describing a mathematical theorem and its proof. What is this poem? Let me know where it can be found. /1/

 \circlearrowright Write an original poem touching on mathematics or its history. [1]

I HOPE THAT YOU ENJOYED THE COURSE. HAVE A GOOD BREAK!