Mathematics-Science 3810H – Ancient and classical mathematics
Trent University, Winter 2022

Take-home Final Examination
Due on Friday, 22 April.

Instructions: Do Parts **I** – **III** and, if you wish, Part **IIII** as well. Give complete answers to receive full credit, including references to any and all sources you used. You may use your texts from this and any other courses, as well as any handouts, class notes, and the like; you may also ask the instructor to clarify the instructions or any of the questions; and you may use a calculator or computer to perform any necessary calculations. *You may not consult any other sources, nor give or receive any other aid on this exam, except with the intructor's explicit permission or as otherwise indicated on a given problem.*

Part I – This and that, and trivia too. Do all three of **1** – **3**.

1. Answer all of **a** – **i**. *[10 = 10×1 each]*

- **a.** Which mathematical tradition used the method of false position?
- **b.** Why is Euclid's Third Postulate necessary?
- **c.** Name someone who wrote out a mathematical work over three thousand years ago.
- **d.** What is the earliest surviving record of numbers?
- **e.** Who first systematically used a symbolic notation for algebraic expressions?
- **f.** Why were reciprocals so important in Mesopotamian mathematics?
- **g.** Which of the number systems mentioned in this course required the fewest distinct symbols and which required the most?
- **h.** Who seems to have been the first to do proofs in mathematics?
- **i.** Name three Greek works touching on mathematics that were each originally made up of 13 books.
- **j.** Who classified music as a branch of mathematics?

2. Compare and contrast the relative strengths and weaknesses of the number systems used in ancient Egypt, Mesopotamia, and Greece. *[15]*

3. Recall Ptolemy's Theorem from class, slightly rephrased here:

> *Suppose A, B, C, and D are any four points on a circle listed in clockwise order. Then $|AB| \cdot |CD| + |AD| \cdot |BC| = |AC| \cdot |BD|$.*

Use Ptolemy's Theorem to prove the following identity for the chord function (in a unit circle, with angles in radians):

$$\text{chord}(\alpha - \beta) = \text{chord}(\alpha)\text{chord}(\pi - \beta) - \text{chord}(\beta)\text{chord}(\pi - \alpha) \quad \textit{[10]}$$

Hint: Make AD a diameter in Ptolemy's Theorem and let α and β be the angles ...

[Parts **II** – **IIII** are on page 2.]

Part II – History. Do *one* of **4** and **5**. You may use additional sources for either one.

4. Read *The architect Kha's protractor* by Amelia Carolina Sparavigna, which you can find at `https://arxiv.org/abs/1107.4946`, plus whatever additional sources you can find about the artifact in question. Summarize the theories about what the artefact is and assess the evidence for and against each theory. *[15]*

5. Find out about and describe the so-called Antikythera Mechanism. What does it tell us about the capabilities of Classical Greek and Hellenistic engineering, astronomy, and mathematics? *[15]*

Part III – Mathematics. Do any *two* of **6** – **8**.

6. Describe how to use the *quadratrix* to trisect an angle and explain in detail why the method works. [*You may consult additional sources for this problem.*] *[10]*

7. Problem 40 of the Rhind Papyrus, translated into modern English and very slightly modified, is:

> *Divide one hundred loaves among five persons in such a way that the shares shall be in arithmetic progression and that one-seventh of the sum of the largest three shares shall be equal to the sum of the smallest two. What are the shares?*

Use the method of false position to solve this problem, showing all the steps. *[10]*

8. Given a line segment of length 1 as a reference, give a detailed ruler and straightedge construction of a line segment of the length of the *golden ratio*, $\varphi = \dfrac{1+\sqrt{5}}{2}$. *[10]*

[Total = 70]

Part IIII - Oldgebra? Bonus!

θ. One of our few clues about the life of Diophantus is a puzzle given in an anthology by a Fifth Century A.D. writer named Metrodorus, given here in a poetic translation:

> 'Here lies Diophantus,' the wonder behold.
> Through art algebraic, the stone tells how old:
> 'God gave him his boyhood one-sixth of his life,
> One twelfth more as youth while whiskers grew rife;
> And then yet one-seventh ere marriage begun;
> In five years there came a bouncing new son.
> Alas, the dear child of master and sage
> After attaining half the measure of his father's life
> chill fate took him.
> After consoling his fate by the science of numbers
> for four years, he ended his life.'

According to the riddle, how old was Diophantus when he died? Please give your reasoning. *[1]*

I HOPE THAT YOU ENJOYED THE COURSE.
NOW ENJOY THE SUMMER, OR ELSE!