

Mathematics 3810H – Ancient and classical mathematics

TRENT UNIVERSITY, Winter 2022

Assignment #6

Pappus' Centroid Theorems

Due on Friday, 8 April.

Pappus of Alexandria (known to be alive *c.* 320 A.D.) was the last great Hellenistic mathematician. His *Collection* is one of our principal sources for who did what in Greek and Hellenistic mathematics, but it also includes a number of original contributions, including the two following results.

THEOREM. If a plane curve is revolved around a line in its plane that does not pass through the curve, then area of the resulting surface of revolution is the product of the length of the curve and the distance traveled by the centroid of the curve.

THEOREM. If a closed plane curve is revolved around a line its plane that does not pass through the curve, then the volume of the resulting solid of revolution is equal to the product of the area of the region bounded by the curve and the distance traveled by the centroid of this region.

1. Use these theorems of Pappus to find the surface area and volume of a torus obtained by revolving a circle of radius r about a line its plane which is a distance $R \geq r$ from the centre of the circle. [4]
2. How might Pappus have gone about – or, if you can find suitable sources, did go about – proving the theorems given above? [6]