

Mathematics 3810H – Ancient and Classical Mathematics

TRENT UNIVERSITY, Winter 2022

Assignment #4

Due on Friday, 11 March.

Along with his theory of proportions, one of the things that Eudoxus of Cnidus is supposed to have developed and used is the method of exhaustion, in which areas are computed by approximating them ever more closely with polygonal shapes. In particular, he is supposed to have used this method to prove that the area of a circle is proportional to the square of its diameter. It is likely that the proof of this fact that is given in Book XII of Euclid's *Elements* is based on Eudoxus' work. The argument given there is based on inscribing regular polygons with 2^n sides in circles; in this assignment you will work through a variation of this argument, one that is made much easier by the use of a little trigonometry.

Given that trigonometry is allowed, you are also allowed to use other modern techniques, including algebra and limits, to answer the questions below.

1. A regular 2^n -gon inscribed in a circle of radius r has area $2^{n-1}r^2 \sin\left(\frac{\pi}{2^{n-1}}\right)$. [3]

Note: As Eudoxus did his work before trigonometric functions were known, so he could not have had this result.

2. Prove that a regular 2^n -gon inscribed in a circle takes up more than $1 - \frac{1}{2^{n-1}}$ of the area of the circle. [4]
3. Use the result in **2** above to prove that the area of a circle is proportional to the square of its radius or diameter. [3]