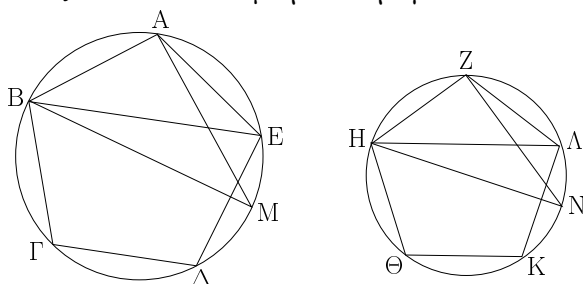


α'.

Τὰ ἐν τοῖς κύκλοις ὅμοια πολύγωνα πρὸς ἀλληλά ἐστὶν ὡς τὰ ἀπὸ τῶν διαμέτρων τετράγωνα.



Ἐστωσαν κύκλοι οἱ  $ABΓ$ ,  $ZHΘ$ , καὶ ἐν αὐτοῖς ὅμοια πολύγωνα ἔστω τὰ  $ABΓΔΕ$ ,  $ZHΘΚΛ$ , διάμετροι δὲ τῶν κύκλων ἔστωσαν  $BM$ ,  $HN$ : λέγω, ὅτι ἐστὶν ὡς τὸ ἀπὸ τῆς  $BM$  τετράγωνον πρὸς τὸ ἀπὸ τῆς  $HN$  τετράγωνον, οὕτως τὸ  $ABΓΔΕ$  πολύγωνον πρὸς τὸ  $ZHΘΚΛ$  πολύγωνον.

Ἐπεζεύχθωσαν γὰρ αἱ  $BE$ ,  $AM$ ,  $ΗΛ$ ,  $ZN$ . καὶ ἐπεὶ ὅμοιον τὸ  $ABΓΔΕ$  πολύγωνον τῷ  $ZHΘΚΛ$  πλουγώνῳ, ἴση ἐστὶ καὶ ἡ ὑπὸ  $BAE$  γωνία τῇ ὑπὸ  $HZΛ$ , καὶ ἐστὶν ὡς ἡ  $BA$  πρὸς τὴν  $AE$ , οὕτως ἡ  $HZ$  πρὸς τὴν  $ZΛ$ . δύο δὴ τρίγωνα ἐστὶ τὰ  $BAE$ ,  $HZΛ$  μίαν γωνίαν μιᾶ γωνία ἴσην ἔχοντα τὴν ὑπὸ  $BAE$  τῇ ὑπὸ  $HZΛ$ , περὶ δὲ τὰς ἴσας γωνίας τὰς πλευρὰς ἀνάλογον ἰσογώνιον ἄρα ἐστὶ τὸ  $ABE$  τρίγωνον τῷ  $ZHL$  τριγώνῳ. ἴση ἄρα ἐστὶν ἡ ὑπὸ  $AEB$  γωνία τῇ ὑπὸ  $ZΛH$ . ἀλλ' ἡ μὲν ὑπὸ  $AEB$  τῇ ὑπὸ  $AMB$  ἐστὶν ἴση· ἐπὶ γὰρ τῆς αὐτῆς περιφερείας βεβήκασιν· ἡ δὲ ὑπὸ  $ZΛH$  τῇ ὑπὸ  $ZNH$ : καὶ ἡ ὑπὸ  $AMB$  ἄρα τῇ ὑπὸ  $ZNH$  ἐστὶν ἴση. ἐστὶ δὲ καὶ ὀρθὴ ἡ ὑπὸ  $BAM$  ὀρθὴ τῇ ὑπὸ  $HZN$  ἴση· καὶ ἡ λοιπὴ ἄρα τῇ λοιπῇ ἐστὶν ἴση. ἰσογώνιον ἄρα ἐστὶ τὸ  $ABM$  τρίγωνον τῷ  $ZHN$  τριγώνῳ. ἀνάλογον ἄρα ἐστὶν ὡς ἡ  $BM$  πρὸς τὴν  $HN$ , οὕτως ἡ  $BA$  πρὸς τὴν  $HZ$ . ἀλλὰ τοῦ μὲν τῆς  $BM$  πρὸς τὴν  $HN$  λόγον διπλασίων ἐστὶν ὁ τοῦ ἀπὸ τῆς  $BM$  τετραγώνου πρὸς τὸ ἀπὸ τῆς  $HN$  τετράγωνον, τοῦ δὲ τῆς  $BA$  πρὸς τὴν  $HZ$  διπλασίων ἐστὶν ὁ τοῦ  $ABΓΔΕ$  πολυγώνου πρὸς τὸ  $ZHΘΚΛ$  πολύγωνον καὶ ὡς ἄρα τὸ ἀπὸ τῆς  $BM$  τετράγωνον πρὸς τὸ ἀπὸ τῆς  $HN$  τετράγωνον, οὕτως τὸ  $ABΓΔΕ$  πολύγωνον πρὸς τὸ  $ZHΘΚΛ$  πολύγωνον.

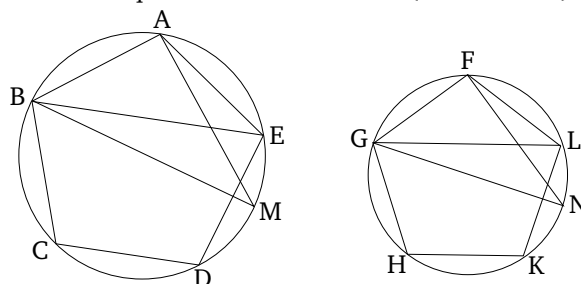
Τὰ ἄρα ἐν τοῖς κύκλοις ὅμοια πολύγωνα πρὸς ἀλληλά ἐστὶν ὡς τὰ ἀπὸ τῶν διαμέτρων τετράγωνα· ὅπερ εἶδει δεῖξαι.

β'.

Οἱ κύκλοι πρὸς ἀλλήλους εἰσὶν ὡς τὰ ἀπὸ τῶν διαμέτρων τετράγωνα.

Proposition 1

Similar polygons (inscribed) in circles are to one another as the squares on the diameters (of the circles).



Let  $ABC$  and  $FGH$  be circles, and let  $ABCDE$  and  $FGHKL$  be similar polygons (inscribed) in them (respectively), and let  $BM$  and  $GN$  be the diameters of the circles (respectively). I say that as the square on  $BM$  is to the square on  $GN$ , so polygon  $ABCDE$  (is) to polygon  $FGHKL$ .

For let  $BE$ ,  $AM$ ,  $GL$ , and  $FN$  have been joined. And since polygon  $ABCDE$  (is) similar to polygon  $FGHKL$ , angle  $BAE$  is also equal to (angle)  $GFL$ , and as  $BA$  is to  $AE$ , so  $GF$  (is) to  $FL$  [Def. 6.1]. So,  $BAE$  and  $GFL$  are two triangles having one angle equal to one angle, (namely),  $BAE$  (equal) to  $GFL$ , and the sides around the equal angles proportional. Triangle  $ABE$  is thus equiangular with triangle  $FGL$  [Prop. 6.6]. Thus, angle  $AEB$  is equal to (angle)  $FLG$ . But,  $AEB$  is equal to  $AMB$ , and  $FLG$  to  $FNG$ , for they stand on the same circumference [Prop. 3.27]. Thus,  $AMB$  is also equal to  $FNG$ . And the right-angle  $BAM$  is also equal to the right-angle  $GFN$  [Prop. 3.31]. Thus, the remaining (angle) is also equal to the remaining (angle) [Prop. 1.32]. Thus, triangle  $ABM$  is equiangular with triangle  $FGN$ . Thus, proportionally, as  $BM$  is to  $GN$ , so  $BA$  (is) to  $GF$  [Prop. 6.4]. But, the (ratio) of the square on  $BM$  to the square on  $GN$  is the square of the ratio of  $BM$  to  $GN$ , and the (ratio) of polygon  $ABCDE$  to polygon  $FGHKL$  is the square of the (ratio) of  $BA$  to  $GF$  [Prop. 6.20]. And, thus, as the square on  $BM$  (is) to the square on  $GN$ , so polygon  $ABCDE$  (is) to polygon  $FGHKL$ .

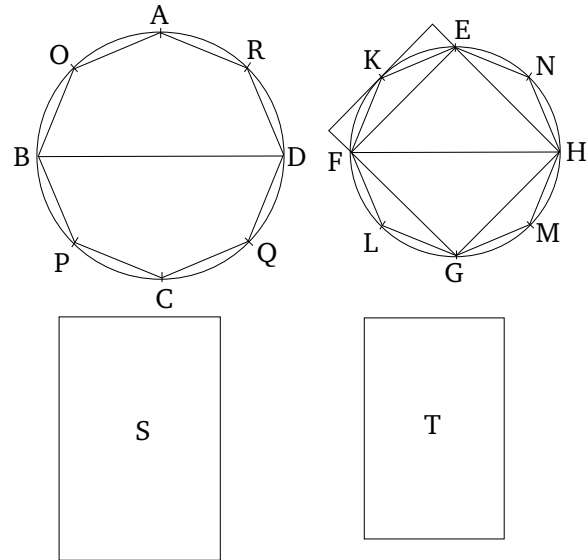
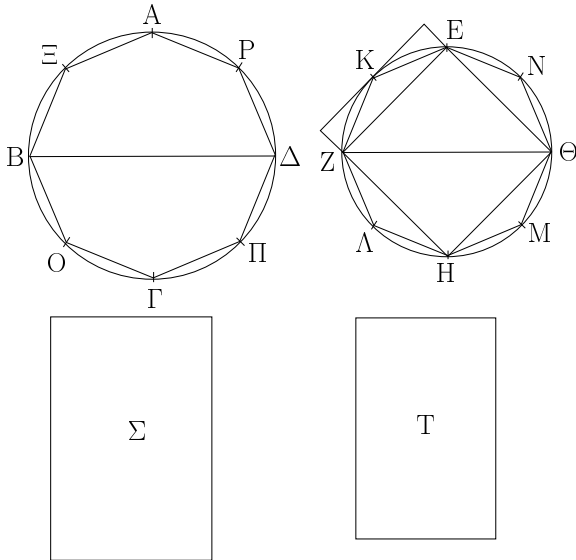
Thus, similar polygons (inscribed) in circles are to one another as the squares on the diameters (of the circles). (Which is) the very thing it was required to show.

Proposition 2

Circles are to one another as the squares on (their) diameters.

Ἐστωσαν κύκλοι οἱ  $AB\Gamma\Delta$ ,  $EZH\Theta$ , διάμετροι δὲ αὐτῶν [ἔστωσαν] αἱ  $B\Delta$ ,  $Z\Theta$ . λέγω, ὅτι ἐστὶν ὡς ὁ  $AB\Gamma\Delta$  κύκλος πρὸς τὸν  $EZH\Theta$  κύκλον, οὕτως τὸ ἀπὸ τῆς  $B\Delta$  τετραγώνου πρὸς τὸ ἀπὸ τῆς  $Z\Theta$  τετραγώνου.

Let  $ABCD$  and  $EFGH$  be circles, and [let]  $BD$  and  $FH$  [be] their diameters. I say that as circle  $ABCD$  is to circle  $EFGH$ , so the square on  $BD$  (is) to the square on  $FH$ .



Εἰ γὰρ μὴ ἐστὶν ὡς ὁ  $AB\Gamma\Delta$  κύκλος πρὸς τὸν  $EZH\Theta$ , οὕτως τὸ ἀπὸ τῆς  $B\Delta$  τετραγώνου πρὸς τὸ ἀπὸ τῆς  $Z\Theta$ , ἔσται ὡς τὸ ἀπὸ τῆς  $B\Delta$  πρὸς τὸ ἀπὸ τῆς  $Z\Theta$ , οὕτως ὁ  $AB\Gamma\Delta$  κύκλος ἢ τοῦ ἐλασσόν τι τοῦ  $EZH\Theta$  κύκλου χωρίον ἢ πρὸς μείζον. ἔστω πρότερον πρὸς ἔλασσον τὸ  $\Sigma$ . καὶ ἐγγεγράφθω εἰς τὸν  $EZH\Theta$  κύκλον τετραγώνου τὸ  $EZH\Theta$ . τὸ δὲ ἐγγεγραμμένον τετραγώνου μείζον ἐστὶν ἢ τὸ ἥμισυ τοῦ  $EZH\Theta$  κύκλου, ἐπειδὴ περὶ ἑὸν διὰ τῶν  $E$ ,  $Z$ ,  $H$ ,  $\Theta$  σημείων ἐφαπτομένης [εὐθείας] τοῦ κύκλου ἀγάγωμεν, τοῦ περιγραφομένου περὶ τὸν κύκλον τετραγώνου ἡμισὺ ἐστὶ τὸ  $EZH\Theta$  τετραγώνου, τοῦ δὲ περιγραφέντος τετραγώνου ἐλάττων ἐστὶν ὁ κύκλος· ὥστε τὸ  $EZH\Theta$  ἐγγεγραμμένον τετραγώνου μείζον ἐστὶ τοῦ ἡμίσεως τοῦ  $EZH\Theta$  κύκλου. τεμήσθωσαν δίχα αἱ  $EZ$ ,  $ZH$ ,  $H\Theta$ ,  $\Theta E$  περιφέρειαι κατὰ τὰ  $K$ ,  $\Lambda$ ,  $M$ ,  $N$  σημεία, καὶ ἐπεζεύχθωσαν αἱ  $EK$ ,  $KZ$ ,  $Z\Lambda$ ,  $\Lambda H$ ,  $H M$ ,  $M\Theta$ ,  $\Theta N$ ,  $NE$ . καὶ ἕκαστον ἄρα τῶν  $EKZ$ ,  $Z\Lambda H$ ,  $H M\Theta$ ,  $\Theta NE$  τριγώνων μείζον ἐστὶν ἢ τὸ ἥμισυ τοῦ καθ' ἑαυτὸ τμήματος τοῦ κύκλου, ἐπειδὴ περὶ ἑὸν διὰ τῶν  $K$ ,  $\Lambda$ ,  $M$ ,  $N$  σημείων ἐφαπτομένης τοῦ κύκλου ἀγάγωμεν καὶ ἀναπληρώσωμεν τὰ ἐπὶ τῶν  $EZ$ ,  $ZH$ ,  $H\Theta$ ,  $\Theta E$  εὐθειῶν παραλληλόγραμμα, ἕκαστον τῶν  $EKZ$ ,  $Z\Lambda H$ ,  $H M\Theta$ ,  $\Theta NE$  τριγώνων ἡμισὺ ἔσται τοῦ καθ' ἑαυτὸ παραλληλογράμμου, ἀλλὰ τὸ καθ' ἑαυτὸ τμήμα ἐλαττόν ἐστὶ τοῦ παραλληλογράμμου· ὥστε ἕκαστον τῶν  $EKZ$ ,  $Z\Lambda H$ ,  $H M\Theta$ ,  $\Theta NE$  τριγώνων μείζον ἐστὶ τοῦ ἡμίσεως τοῦ καθ' ἑαυτὸ τμήματος τοῦ κύκλου. τέμνοντες δὴ τὰς ὑπολει-

For if the circle  $ABCD$  is not to the (circle)  $EFGH$ , as the square on  $BD$  (is) to the (square) on  $FH$ , then as the (square) on  $BD$  (is) to the (square) on  $FH$ , so circle  $ABCD$  will be to some area either less than, or greater than, circle  $EFGH$ . Let it, first of all, be (in that ratio) to (some) lesser (area),  $S$ . And let the square  $EFGH$  have been inscribed in circle  $EFGH$  [Prop. 4.6]. So the inscribed square is greater than half of circle  $EFGH$ , inasmuch as if we draw tangents to the circle through the points  $E$ ,  $F$ ,  $G$ , and  $H$ , then square  $EFGH$  is half of the square circumscribed about the circle [Prop. 1.47], and the circle is less than the circumscribed square. Hence, the inscribed square  $EFGH$  is greater than half of circle  $EFGH$ . Let the circumferences  $EF$ ,  $FG$ ,  $GH$ , and  $HE$  have been cut in half at points  $K$ ,  $L$ ,  $M$ , and  $N$  (respectively), and let  $EK$ ,  $KF$ ,  $FL$ ,  $LG$ ,  $GM$ ,  $MH$ ,  $HN$ , and  $NE$  have been joined. And, thus, each of the triangles  $EKF$ ,  $FLG$ ,  $GMH$ , and  $HNE$  is greater than half of the segment of the circle about it, inasmuch as if we draw tangents to the circle through points  $K$ ,  $L$ ,  $M$ , and  $N$ , and complete the parallelograms on the straight-lines  $EF$ ,  $FG$ ,  $GH$ , and  $HE$ , then each of the triangles  $EKF$ ,  $FLG$ ,  $GMH$ , and  $HNE$  will be half of the parallelogram about it, but the segment about it is less than the parallelogram. Hence, each of the triangles  $EKF$ ,  $FLG$ ,  $GMH$ , and  $HNE$  is greater than half of the segment of the circle about it. So, by cutting the circumferences remaining behind in half, and joining

πομένας περιφερείας δίχα καὶ ἐπιζευγύντες εὐθείας καὶ τοῦτο αἰεὶ ποιοῦντες καταλείβομεν τινα ἀποτμήματα τοῦ κύκλου, ἃ ἔσται ἐλάσσοντα τῆς ὑπεροχῆς, ἣ ὑπερέχει ὁ ΕΖΗΘ κύκλος τοῦ Σ χωρίου. ἐδείχθη γὰρ ἐν τῷ πρώτῳ θεωρήματι τοῦ δεκάτου βιβλίου, ὅτι δύο μεγεθῶν ἀνίσων ἐκκειμένων, ἐὰν ἀπὸ τοῦ μείζονος ἀφαιρηθῇ μείζον ἢ τὸ ἥμισυ καὶ τοῦ καταλειπομένου μείζον ἢ τὸ ἥμισυ, καὶ τοῦτο αἰεὶ γίγνηται, λειφθήσεται τι μέγεθος, ὃ ἔσται ἔλασσον τοῦ ἐκκειμένου ἐλάσσονος μεγέθους. λελείφθω οὖν, καὶ ἔστω τὰ ἐπὶ τῶν ΕΚ, ΚΖ, ΖΛ, ΛΗ, ΗΜ, ΜΘ, ΘΝ, ΝΕ τμήματα τοῦ ΕΖΗΘ κύκλου ἐλάττονα τῆς ὑπεροχῆς, ἣ ὑπερέχει ὁ ΕΖΗΘ κύκλος τοῦ Σ χωρίου. λοιπὸν ἄρα τὸ ΕΚΖΛΗΜΘΝ πολύγωνον μείζον ἔστι τοῦ Σ χωρίου. ἐγγεγράφθω καὶ εἰς τὸν ΑΒΓΔ κύκλον τῷ ΕΚΖΛΗΜΘΝ πολυγώνῳ ὅμοιον πολύγωνον τὸ ΑΞΒΟΓΠΔΡ· ἔστιν ἄρα ὡς τὸ ἀπὸ τῆς ΒΔ τετράγωνον πρὸς τὸ ἀπὸ τῆς ΖΘ τετράγωνον, οὕτως τὸ ΑΞΒΟΓΠΔΡ πολύγωνον πρὸς τὸ ΕΚΖΛΗΜΘΝ πολύγωνον. ἀλλὰ καὶ ὡς τὸ ἀπὸ τῆς ΒΔ τετράγωνον πρὸς τὸ ἀπὸ τῆς ΖΘ, οὕτως ὁ ΑΒΓΔ κύκλος πρὸς τὸ Σ χωρίον· καὶ ὡς ἄρα ὁ ΑΒΓΔ κύκλος πρὸς τὸ Σ χωρίον, οὕτως τὸ ΑΞΒΟΓΠΔΡ πολύγωνον πρὸς τὸ ΕΚΖΛΗΜΘΝ πολύγωνον· ἐναλλάξ ἄρα ὡς ὁ ΑΒΓΔ κύκλος πρὸς τὸ ἐν αὐτῷ πολυγώνῳ, οὕτως τὸ Σ χωρίον πρὸς τὸ ΕΚΖΛΗΜΘΝ πολύγωνον. μείζων δὲ ὁ ΑΒΓΔ κύκλος τοῦ ἐν αὐτῷ πολυγώνου· μείζων ἄρα καὶ τὸ Σ χωρίον τοῦ ΕΚΖΛΗΜΘΝ πολυγώνου. ἀλλὰ καὶ ἔλαττον· ὅπερ ἔστιν ἀδύνατον. οὐκ ἄρα ἔστιν ὡς τὸ ἀπὸ τῆς ΒΔ τετράγωνον πρὸς τὸ ἀπὸ τῆς ΖΘ, οὕτως ὁ ΑΒΓΔ κύκλος πρὸς ἔλασσόν τι τοῦ ΕΖΗΘ κύκλου χωρίου. ὁμοίως δὴ δεῖξομεν, ὅτι οὐδὲ ὡς τὸ ἀπὸ ΖΘ πρὸς τὸ ἀπὸ ΒΔ, οὕτως ὁ ΕΖΗΘ κύκλος πρὸς ἔλασσόν τι τοῦ ΑΒΓΔ κύκλου χωρίου.

Λέγω δὴ, ὅτι οὐδὲ ὡς τὸ ἀπὸ τῆς ΒΔ πρὸς τὸ ἀπὸ τῆς ΖΘ, οὕτως ὁ ΑΒΓΔ κύκλος πρὸς μείζον τι τοῦ ΕΖΗΘ κύκλου χωρίου.

Εἰ γὰρ δυνατόν, ἔστω πρὸς μείζον τὸ Σ. ἀνάπαλιν ἄρα [ἔστιν] ὡς τὸ ἀπὸ τῆς ΖΘ τετράγωνον πρὸς τὸ ἀπὸ τῆς ΔΒ, οὕτως τὸ Σ χωρίον πρὸς τὸν ΑΒΓΔ κύκλον. ἀλλ' ὡς τὸ Σ χωρίον πρὸς τὸν ΑΒΓΔ κύκλον, οὕτως ὁ ΕΖΗΘ κύκλος πρὸς ἔλαττόν τι τοῦ ΑΒΓΔ κύκλου χωρίου· καὶ ὡς ἄρα τὸ ἀπὸ τῆς ΖΘ πρὸς τὸ ἀπὸ τῆς ΒΔ, οὕτως ὁ ΕΖΗΘ κύκλος πρὸς ἔλασσόν τι τοῦ ΑΒΓΔ κύκλου χωρίου· ὅπερ ἀδύνατον ἐδείχθη. οὐκ ἄρα ἔστιν ὡς τὸ ἀπὸ τῆς ΒΔ τετράγωνον πρὸς τὸ ἀπὸ τῆς ΖΘ, οὕτως ὁ ΑΒΓΔ κύκλος πρὸς μείζον τι τοῦ ΕΖΗΘ κύκλου χωρίου. ἐδείχθη δέ, ὅτι οὐδὲ πρὸς ἔλασσον ἔστιν ἄρα ὡς τὸ ἀπὸ τῆς ΒΔ τετράγωνον πρὸς τὸ ἀπὸ τῆς ΖΘ, οὕτως ὁ ΑΒΓΔ κύκλος πρὸς τὸν ΕΖΗΘ κύκλον.

Οἱ ἄρα κύκλοι πρὸς ἀλλήλους εἰσὶν ὡς τὰ ἀπὸ τῶν

straight-lines, and doing this continually, we will (eventually) leave behind some segments of the circle whose (sum) will be less than the excess by which circle  $EFGH$  exceeds the area  $S$ . For we showed in the first theorem of the tenth book that if two unequal magnitudes are laid out, and if (a part) greater than a half is subtracted from the greater, and (if from) the remainder (a part) greater than a half (is subtracted), and this happens continually, then some magnitude will (eventually) be left which will be less than the lesser laid out magnitude [Prop. 10.1]. Therefore, let the (segments) have been left, and let the (sum of the) segments of the circle  $EFGH$  on  $EK, KF, FL, LG, GM, MH, HN$ , and  $NE$  be less than the excess by which circle  $EFGH$  exceeds area  $S$ . Thus, the remaining polygon  $EKFLGMHN$  is greater than area  $S$ . And let the polygon  $AOBPCQDR$ , similar to the polygon  $EKFLGMHN$ , have been inscribed in circle  $ABCD$ . Thus, as the square on  $BD$  is to the square on  $FH$ , so polygon  $AOBPCQDR$  (is) to polygon  $EKFLGMHN$  [Prop. 12.1]. But, also, as the square on  $BD$  (is) to the square on  $FH$ , so circle  $ABCD$  (is) to area  $S$ . And, thus, as circle  $ABCD$  (is) to area  $S$ , so polygon  $AOBPCQDR$  (is) to polygon  $EKFLGMHN$  [Prop. 5.11]. Thus, alternately, as circle  $ABCD$  (is) to the polygon (inscribed) within it, so area  $S$  (is) to polygon  $EKFLGMHN$  [Prop. 5.16]. And circle  $ABCD$  (is) greater than the polygon (inscribed) within it. Thus, area  $S$  is also greater than polygon  $EKFLGMHN$ . But, (it is) also less. The very thing is impossible. Thus, the square on  $BD$  is not to the (square) on  $FH$ , as circle  $ABCD$  (is) to some area less than circle  $EFGH$ . So, similarly, we can show that the (square) on  $FH$  (is) not to the (square) on  $BD$  as circle  $EFGH$  (is) to some area less than circle  $ABCD$  either.

So, I say that neither (is) the (square) on  $BD$  to the (square) on  $FH$ , as circle  $ABCD$  (is) to some area greater than circle  $EFGH$ .

For, if possible, let it be (in that ratio) to (some) greater (area),  $S$ . Thus, inversely, as the square on  $FH$  [is] to the (square) on  $DB$ , so area  $S$  (is) to circle  $ABCD$  [Prop. 5.7 corr.]. But, as area  $S$  (is) to circle  $ABCD$ , so circle  $EFGH$  (is) to some area less than circle  $ABCD$  (see lemma). And, thus, as the (square) on  $FH$  (is) to the (square) on  $BD$ , so circle  $EFGH$  (is) to some area less than circle  $ABCD$  [Prop. 5.11]. The very thing was shown (to be) impossible. Thus, as the square on  $BD$  is to the (square) on  $FH$ , so circle  $ABCD$  (is) not to some area greater than circle  $EFGH$ . And it was shown that neither (is it in that ratio) to (some) lesser (area). Thus, as the square on  $BD$  is to the (square) on  $FH$ , so circle  $ABCD$  (is) to circle  $EFGH$ .

διαμέτρων τετραγώνων· ὅπερ ἔδει δεῖξαι.

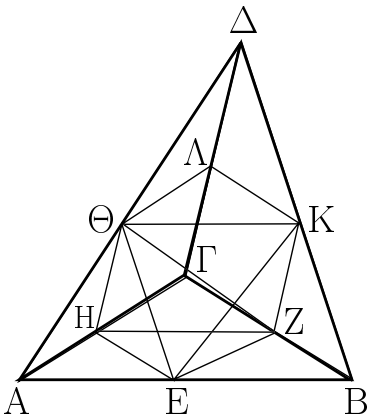
Λήμμα.

Λέγω δὴ, ὅτι τοῦ Σ χωρίου μείζονος ὄντος τοῦ ΕΖΗΘ κύκλου ἐστὶν ὡς τὸ Σ χωρίον πρὸς τὸν ΑΒΓΔ κύκλον, οὕτως ὁ ΕΖΗΘ κύκλος πρὸς ἕλαττόν τι τοῦ ΑΒΓΔ κύκλου χωρίου.

Γεγονέτω γὰρ ὡς τὸ Σ χωρίον πρὸς τὸν ΑΒΓΔ κύκλον, οὕτως ὁ ΕΖΗΘ κύκλος πρὸς τὸ Τ χωρίον. λέγω, ὅτι ἕλαττόν ἐστι τὸ Τ χωρίον τοῦ ΑΒΓΔ κύκλου. ἐπεὶ γὰρ ἐστὶν ὡς τὸ Σ χωρίον πρὸς τὸν ΑΒΓΔ κύκλον, οὕτως ὁ ΕΖΗΘ κύκλος πρὸς τὸ Τ χωρίον, ἐναλλάξ ἐστὶν ὡς τὸ Σ χωρίον πρὸς τὸν ΕΖΗΘ κύκλον, οὕτως ὁ ΑΒΓΔ κύκλος πρὸς τὸ Τ χωρίον. μείζον δὲ τὸ Σ χωρίον τοῦ ΕΖΗΘ κύκλου· μείζων ἄρα καὶ ὁ ΑΒΓΔ κύκλος τοῦ Τ χωρίου. ὥστε ἐστὶν ὡς τὸ Σ χωρίον πρὸς τὸν ΑΒΓΔ κύκλον, οὕτως ὁ ΕΖΗΘ κύκλος πρὸς ἕλαττόν τι τοῦ ΑΒΓΔ κύκλου χωρίου· ὅπερ ἔδει δεῖξαι.

γ'.

Πᾶσα πυραμὶς τρίγωνον ἔχουσα βάσιν διαιρεῖται εἰς δύο πυραμίδας ἴσας τε καὶ ὁμοίας ἀλλήλαις καὶ [ὁμοίας] τῇ ὅλῃ τριγώνου ἔχουσας βάσεις καὶ εἰς δύο πρίσματα ἴσα· καὶ τὰ δύο πρίσματα μείζονά ἐστιν ἢ τὸ ἥμισυ τῆς ὅλης πυραμίδος.



Ἐστω πυραμὶς, ἥς βάσις μὲν ἐστὶ τὸ ΑΒΓ τρίγωνον, κορυφὴ δὲ τὸ Δ σημεῖον· λέγω, ὅτι ἡ ΑΒΓΔ πυραμὶς διαιρεῖται εἰς δύο πυραμίδας ἴσας ἀλλήλαις τριγώνου βάσεις ἔχούσας καὶ ὁμοίας τῇ ὅλῃ καὶ εἰς δύο πρίσματα ἴσα· καὶ τὰ δύο πρίσματα μείζονά ἐστιν ἢ τὸ ἥμισυ τῆς ὅλης πυραμίδος.

Τετμήσθωσαν γὰρ αἱ ΑΒ, ΒΓ, ΓΑ, ΑΔ, ΔΒ, ΔΓ δίχα κατὰ τὰ Ε, Ζ, Η, Θ, Κ, Λ σημεῖα, καὶ ἐπεζεύχθωσαν αἱ ΘΕ, ΕΗ, ΗΘ, ΘΚ, ΚΛ, ΛΘ, ΚΖ, ΖΗ. ἐπεὶ ἴση

Thus, circles are to one another as the squares on (their) diameters. (Which is) the very thing it was required to show.

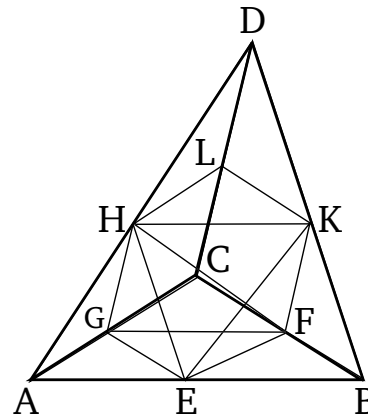
Lemma

So, I say that, area  $S$  being greater than circle  $EFGH$ , as area  $S$  is to circle  $ABCD$ , so circle  $EFGH$  (is) to some area less than circle  $ABCD$ .

For let it have been contrived that as area  $S$  (is) to circle  $ABCD$ , so circle  $EFGH$  (is) to area  $T$ . I say that area  $T$  is less than circle  $ABCD$ . For since as area  $S$  is to circle  $ABCD$ , so circle  $EFGH$  (is) to area  $T$ , alternately, as area  $S$  is to circle  $EFGH$ , so circle  $ABCD$  (is) to area  $T$  [Prop. 5.16]. And area  $S$  (is) greater than circle  $EFGH$ . Thus, circle  $ABCD$  (is) also greater than area  $T$  [Prop. 5.14]. Hence, as area  $S$  is to circle  $ABCD$ , so circle  $EFGH$  (is) to some area less than circle  $ABCD$ . (Which is) the very thing it was required to show.

Proposition 3

Any pyramid having a triangular base is divided into two pyramids having triangular bases (which are) equal, similar to one another, and [similar] to the whole, and into two equal prisms. And the (sum of the) two prisms is greater than half of the whole pyramid.



Let there be a pyramid whose base is triangle  $ABC$ , and (whose) apex (is) point  $D$ . I say that pyramid  $ABCD$  is divided into two pyramids having triangular bases (which are) equal to one another, and similar to the whole, and into two equal prisms. And the (sum of the) two prisms is greater than half of the whole pyramid.

For let  $AB$ ,  $BC$ ,  $CA$ ,  $AD$ ,  $DB$ , and  $DC$  have been cut in half at points  $E$ ,  $F$ ,  $G$ ,  $H$ ,  $K$ , and  $L$  (respectively). And let  $HE$ ,  $EG$ ,  $GH$ ,  $HK$ ,  $KL$ ,  $LH$ ,  $KF$ , and  $FG$  have