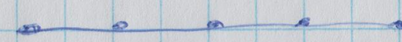
# Ancient Egyptian geometry

Some Greeks (at least) believed that the Egyptians invented geometry, specifically for surveying. This is the assertion made in Herodotus' Histories. Is this plausible or likely to be correct?

Maybe... surveying for constructing buildings, or canals, or monuments, or even to determine area of a plot (for tax purposes), require some practical knowledge of geometry.

eg How do you lay out a square-based pyramid? Measuring length is not so hard - mark of a suitable length of rope in equal increments. ≡



How do you measure out the right angles?

## Herodotus on the origins of Egyptian geometry

The Greek historian Herodotus (*c.* 485-425 B.C.) asserted that the Egyptians invented geometry to deal with surveying problems (see §1-2 of the text). The relevant passage from Herodotus' *Histories*, as translated by G.C. Macaulay (from an edition published in 1890), is:

> 108. Then Sesostris, having returned to Egypt and having taken vengeance on his brother, employed the multitude which he had brought in of those whose lands he had subdued, as follows: these were they who drew the stones which in the reign of this king were brought to the temple of Hephaistos, being of very great size; and also these were compelled to dig all the channels which now are in Egypt; and thus (having no such purpose) they caused Egypt, which before was all fit for riding and driving, to be no longer fit for this from thenceforth: for from that time forward Egypt, though it is plain land, has become all unfit for riding and driving, and the cause has been these channels, which are many and run in all directions. But the reason why the king cut up the land was this, namely because those of the Egyptians who had their cities not on the river but in the middle of the country, being in want of water when the river went down from them, found their drink brackish because they had it from wells.
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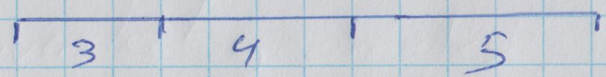
> 109. For this reason Egypt was cut up; and they said that this king distributed the land to all the Egyptians, giving an equal square portion to each man, and from this he made his revenue, having appointed them to pay a certain rent every year: and if the river should take away anything from any man's portion, he would come to the king and declare that which had happened, and the king used to send men to examine and to find out by measurement how much less the piece of land had become, in order that for the future the man might pay less, in proportion to the rent appointed: and I think that thus the art of geometry was found out and afterwards came into Hellas also. For as touching the sun-dial and the gnomon and the twelve divisions of the day, they were learnt by the Hellenes from the Babylonians.

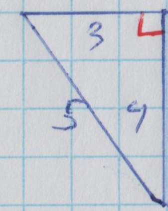
**Questions.** Having read this passage, do you think Herodotus' belief about the origins of Egyptian geometry is likely to be true? Can you think of any other hypotheses that fit the available evidence? What other evidence, one way or the other, do we have or could acquire?

Keep in mind that Herodotus was not the most reliable historian by our standards: he would tell a good story even if it was inconsistent with other stories he'd already told. Mind you, by his own account, he simply reported on what he saw and what he was told. Herodotus would have been rather more familiar with Egypt than most Greeks, having travelled there before writing his *Histories*.

At any rate, no Pharaoh conquered the then-known world, as Herodotus relates Sesostris did. The name Sesostris may be a Hellenization of Senusret or Senwosret, a name used by several Pharaohs of the Twelfth Dynasty of the Middle Kingdom. In particular, Senusret III was a conqueror (on a more modest scale than Sesostris) and set up some monuments as memorials to his campaigns similar to those Herodotus described Sesostris as having put up.

The Egyptians seem to have done this by marking out a rope in proportions 3, 4, 5.




If you connect the ends and stretch out the rope while holding at the marks you make a 3-4-5 triangle giving a right angle at the mark between 3 & 4.

($3^2+4^2=5^2$ so you get a right angle by the converse of the Pythagorean theorem)

So it is at least plausible that the Egyptians invented geometry to solve practical problems like this. What we know from surviving texts is largely focussed on "practical" problems.

eg In one papyrus we get the computation of the area of a circular field. The calculation of a field of diameter $d$ is given as $\left(d - \frac{d}{9}\right)^2 = \left(\frac{8}{9}d\right)^2 = \frac{64}{81}d^2$.

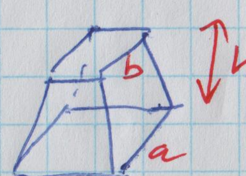
Our formula is $\pi r^2 = \pi\left(\frac{d}{2}\right)^2 = \frac{\pi d^2}{4}$, which is not too far off...

$\frac{64}{81} \approx 0.7901 \qquad \frac{\pi}{4} \approx 0.7854$

Here the interest is in getting a result accurate for practical purposes.

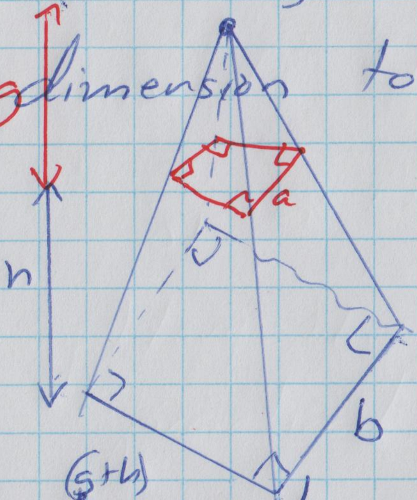
In the Moscow papyrus there is a computation of the volume of a truncated pyramid.

We know from the Ahmose papyrus that the Egyptians knew that the volume of a pyramid with base area $B$ and height $h$ is $\frac{1}{3}hB$.




that gives the correct result for the given values of $a, b, h$. How did they know?

For us, we'd derive the formula by relating the dimension to algebraic expressions and manipulating these. (4)




The truncated pyramid subtracts the part above the red cross-section

Let $g$ be the height of the part removed & $h$ be the height of the remainder, $a$ is the side length at height $h$ & $b$ is the side length at the base.

$g : (g+h)$ as $a : b$

$\Rightarrow g = \frac{a}{b}(g+h)$

$V_{entire} = \frac{1}{3} h^{eight} \cdot B = \frac{1}{3}(h+g) b^2$

$V_{removed} = \frac{1}{3} g \cdot B = \frac{1}{3} g \cdot a^2$

$V_{truncated} = V_{entire} - V_{removed} = \frac{1}{3}(h+g) b^2 - \frac{1}{3} g a^2$

$\Rightarrow g(1 - \frac{a}{b}) = \frac{ah}{b}$

$\Rightarrow g = \frac{ah/b}{1 - \frac{a}{b}} = \frac{ah}{b-a}$

$= \frac{1}{3}\left(h + \frac{ah}{b-a}\right) b^2 - \frac{1}{3}\frac{ah}{b-a} a^2$

$= \frac{1}{3} h b^2 + \frac{1}{3}\frac{b^2 ah}{b-a} - \frac{1}{3}\frac{ah}{b-a} \cdot a^2$

$= \frac{1}{3} h b^2 + \frac{1}{3}(b^2 - a^2)\frac{ah}{b-a}$

$$= \frac{1}{3}hb^2 + \frac{1}{3}\cancel{(b-a)}(b+a)\frac{ah}{\cancel{b-a}}$$

$$= \frac{1}{3}h(b^2 + ab + a^2)$$

which is the correct formula

... that gives the same answers as the procedure given in the Moscow papyrus.

There is no clue given as to how they got this procedure. As far as we know, there is no real idea of generalization or proof as we know it.