Egyptian fractions, division, and some algebra

... but first, a word about our sources.

These are mostly fragmentary pieces of papyrus, since inscriptions on monuments & tombs don't show calculations. The most complete & comprensive is the so-called Rhind or Ahmose papyrus:

- writted down by a scribe named Ahmose (or Ahmes) c 1650 B.C. [during the Second Intermediate Period, under a Hyksos monarch)
- he says he copied a work that had been written a couple of centuries before that
- mostly a collection of problems and their solutions
  - mostly practical, do not describe general techniques except by example
  - some problems, eg involving summing a finite geometric series, that have no apparent pratical application.
- It's mostly in the "hieratic" script rather than in formal hieroghyphics

Fractions: Egyptians mostly used unit fractions, ie $\frac{1}{n}$ for some n, and used the symbol $\frown$ to indicate a fraction

eg $\frac{1}{12}$ would be written as $\overset{\frown}{\cap\, \text{II}}$

A small number of exceptions:

$\frac{1}{2}$ ⊏

$\frac{1}{4}$ X

$\frac{2}{3}$ $\overset{\frown}{\text{A}}$

Other than two thirds, Egyptians broke fractions down into sums of unit fractions:

eg $\frac{3}{5}$ would be written as $\overset{\frown}{\text{II}}\ \overset{\frown}{\cap}$, ie $\frac{1}{2}+\frac{1}{10}$

$= \frac{5}{10}+\frac{1}{10} = \frac{6}{10} = \frac{3}{5}$

Q: How did they do the breakdown?

A: In general, this is not clear:

Most often, they seem to have expanded a fraction by fitting the largest unit fraction into what was left:

eg $\frac{3}{7} = \frac{1}{4} + \frac{1}{6} + \frac{1}{41}$

$$\frac{3}{7} - \frac{1}{4} = \frac{12}{28} - \frac{7}{28} = \frac{5}{28}$$

$$\frac{5}{28} - \frac{1}{6} = \frac{30}{84} - \frac{28}{84} = \frac{2}{84} = \frac{1}{41}$$

To help with whatever process they used they used (at least in the Ahmose papyrus) a table of breakdowns of $\frac{2}{n}$ fractions into unit fractions.

The Egyptians adapted their doubling & adding multiplication algorithm to dividing with the help of halving (or thirding or tenthing) as an additional operation.

Problem 25 from the Ahmose papyrus:

Find the quantity of 21 divided by 4.

"Calculate with 4 to get 21."

using modern numbers

	1	4	✓
double	2	8	
double	4	16	✓
halve the original	$\frac{1}{2}$	2	
halve again	$\frac{1}{4}$	1	✓
Total:		21	

[if we doubled again, we'ed exceed 21]

Adding up the corresponding ones from the first column gives us $1+4+\frac{1}{4}=5\frac{1}{4}=\frac{21}{4}$.

Ahmose goes on to check the answer by multiplying his answer by 4 to ensure that one does get 21.

How did the Egyptians solve some linear equations?

Another problem from the Rhind / Ahmose papyrus:

Problem ? : "A quantity and its half added together make 16. What is the quantity?"

(An "aha" problem.)

The Egyptian word for "quantity" or "heap" was "aha".

In our terms this is solving $x + \frac{x}{2} = 16$.

"Assume [the quantity is] 2".

1	2
$\frac{1}{2}$	1
Total	3

"As many times as 3 must be multiplied to make 16, so many times 2 will give the quantity."

[ie compute $16 \div 3$ ...]

	1	3 ✓
double	2	6
double	4	12 ✓
third ~~[illegible]~~	$\frac{1}{3}$	1 ✓
Total		16 " 3+12+1

[doubling again would get us over 16]

So the ~~quantity~~ multiple needed is $1 + 4 + \frac{1}{3} = 5\frac{1}{3} = \frac{16}{3}$.

(Again, Ahmose goes on to check his answer.)

Thus the correct answer is 2 times $\frac{16}{3}$,
which Ahmose computes as usual to get $10\frac{2}{3}$. ($\cap$ A)
(& then checks the answer.)

This use of 2 as an initial assumption:
it's false, but it leads to a solution
is why the method is referred to as
the method of (a single) false position.

Next: a brief look at geometry in
Egypt.