Mathematics-Science 3810H – Ancient and classical mathematics
Trent University, Winter 2020

Take-home Final Examination
Due by email on Friday, 17 April.

Instructions: Give complete answers to receive full credit, including references to any and all sources you used. You may use your texts from this and any other courses, as well as any handouts, class notes, and the like; you may also ask the instructor to clarify the instructions or any of the questions; and you may use a calculator or computer to perform any necessary calculations. *You may not consult any other sources, nor give or receive any other aid on this exam, except with the instructor's explicit permission or as otherwise indicated on a given problem.*

Part I – Trivia and this and that. Do *all* four of **1** – **4**.

1. Answer all of **a** – **i**. *[10 = 10 × 1 each]*

- **a.** Who is supposed to have been the first to give proofs of theorems?
- **b.** Name three classical mathematicians who were also astronomers.
- **c.** Name an Egyptian or Mesopotamian scribe who wrote about mathematics.
- **d.** Who invented the method of exhaustion?
- **e.** What ancient number system mentioned in this course uses the fewest distinct symbols?
- **f.** Give three approximations to π used before 450 A.D.
- **g.** Name two Greek mathematicians who wrote about music.
- **h.** Name an ancient number system that we still commonly use parts of for specific tasks.
- **i.** Name the mathematician who wrote the definitive text on astrology.
- **j.** Name a Greek mathematician who also wrote poems.

2. Compare and contrast the relative strengths and weaknesses of the number systems used in ancient Egypt, Mesopotamia, and Greece. *[15]*

3. Recall Ptolemy's Theorem from class:

> *Suppose A, B, C, and D are any four points on a circle listed in clockwise order. Then $|AB| \cdot |CD| + |AD| \cdot |BC| = |AC| \cdot |BD|$.*

Use Ptolemy's Theorem to prove the following identity for the chord function (in a unit circle, with angles in radians):

$$\text{chord}(\alpha - \beta) = \text{chord}(\alpha)\text{chord}(\pi - \beta) - \text{chord}(\beta)\text{chord}(\pi - \alpha) \quad \textit{[10]}$$

Hint: Make AD a diameter in Ptolemy's Theorem and let α and β be the angles ...

4. Titus Flavius Josephus/Joseph ben Matthias was a Roman-Jewish historian who is best known for *The Jewish War*, a history of the Jewish revolt against Rome 66–70 A.D., in which he participated on both sides. An incident mentioned *The Jewish War* gave rise to what is now called the Josephus problem in discrete mathematics. *[You may consult any sources you like to help answer* **a** *and* **b** *below.]*

- **a.** Describe the Josephus problem. *[5]*
- **b.** Describe the incident that gave rise to the Josephus problem. *[5]*

Part II – History. Do *one* (1) of **5** and **6**.

5. "The feature distinguishing mathematics from the natural sciences is that its ultimate validating principle is proof rather than experience." Does the development and evolution of geometry in ancient and classical times tend support this assertion or not? *[You may consult any sources you like to help answer this.] [15]*

6. Briefly describe Plato's influence on mathematics and argue whether his influence was beneficial or not to the development of mathematics. *[You may consult any sources you like to help answer this.] [15]*

Part III – Mathematics. Do any *two* (2) of **7** – **9**.

7. Do both of **a** and **b**.

a. Suppose n and $2^n - 1$ are both prime. Show that $2^{n-1}(2^n - 1)$ is a perfect number. *[7]*

b. Would $2^{n-1}(2^n - 1)$ still have to be a perfect number if n were not prime? Why or why not? *[3]*

8. If u and v are two numbers and $u \geq v$, their *average* is $(u+v)/2$ and their *semidifference* is $(u - v)/2$. Do all three of **a** – **c**.

a. Express uv in terms of the average and semidifference of u and v. *[4]*

b. Given that $a = u + v$ and $b = uv$, solve for u and v in terms of a and b. *[2]*

c. How can the method in **b** be used to solve quadratic equations? *[4]*

9. Starting with a line segment of length 1, give a detailed ruler and compass construction of a line segment of length $\sqrt{5}$. *[You may consult any sources you like to help answer this.] [10]*

[Total = 80]

Part IV – Bonus. For fun *and* marks!

$\wedge\ \wedge$
$\wedge\ \wedge$**|.** Write a poem touching on mathematics or its history. *[1]*

$\wedge\ \wedge$
$\wedge\ \wedge$**||.** Answer the riddle below, which supposedly gives the length of Diophantus' life. *[1]*

126.—ΑΑΛΟ

Οὗτός τοι Διόφαντον ἔχει τάφος· ἆ μέγα θαῦμα·
καὶ τάφος ἐκ τέχνης μέτρα βίοιο λέγει.
ἔκτην κουρίζειν βιότου θεὸς ὥπασε μοίρην·
δωδεκάτην δ' ἐπιθείς, μῆλα πόρεν χνοάειν·
τῇ δ' ἄρ' ἐφ' ἑβδομάτῃ τὸ γαμήλιον ἤψατο φέγγος, 5
ἐκ δὲ γάμων πέμπτῳ παῖδ' ἐπένευσεν ἔτει.
αἰαῖ, τηλύγετον δειλὸν τέκος, ἥμισυ πατρὸς
†τοῦδε καὶ ἡ κρυερὸς μέτρον ἐλὼν βιότου.
πένθος δ' αὖ πισύρεσσι παρηγορέων ἐνιαυτοῖς
τῇδε πύσου σοφίῃ τέρμ' ἐπέρησε βίου. 10

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This tomb holds Diophantus. Ah, how great a marvel! the tomb tells scientifically the measure of his life. God granted him to be a boy for the sixth part of his life, and adding a twelfth part to this, he clothed his cheeks with down; He lit him the light of wedlock after a seventh part, and five years after his marriage He granted him a son. Alas! late-born wretched child; after attaining the measure of half his father's life, chill Fate took him. After consoling his grief by this science of numbers for four years he ended his life.

Metrodorus, Epigram 126, Greek Anthology

I HOPE THAT YOU ENJOYED THE COURSE!
HAVE A GOOD SUMMER!