ΒΘ σύμμετρός ἐστιν. καὶ ἐπεὶ πενταπλάσιόν ἐστι τὸ ἀπὸ τῆς ΒΚ τοῦ ἀπὸ τῆς ΚΜ, τὸ ἄρα ἀπὸ τῆς ΒΚ πρὸς τὸ ἀπὸ τὴς ΚΜ λόγον ἔχει, ὃν ε΄ πρὸς ἕν. ἀναστρέψαντι ἄρα τὸ ἀπὸ τῆς ΒΚ πρὸς τὸ ἀπὸ τῆς Ν λόγον ἔχει, ὃν ε΄ πρὸς δ΄, οὐχ ὃν τετράγωνος πρὸς τετράγωνον· ἀσύμμετρος ἄρα ἐστὶν ἡ ΒΚ τῇ Ν· ἡ ΒΚ ἄρα τῆς ΚΜ μεῖζον δύναται τῷ ἀπὸ ἀσυμμέτρου ἑαυτῇ. ἐπεὶ οὖν ὅλη ἡ ΒΚ τῆς προσαρμοζούσης τῆς ΚΜ μεῖζον δύναται τῷ ἀπὸ ἀσυμμέτρου ἑαυτῇ, καὶ ὅλη ἡ ΒΚ σύμμετρός ἐστι τῇ ἐκκειμένῃ ῥητῇ τῇ ΒΘ, ἀποτομὴ ἄρα τετάρτη ἐστὶν ἡ ΜΒ. τὸ δὲ ὑπὸ ῥητῆς καὶ ἀποτομῆς τετάρτης περιεχόμενον ὀρθογώνιον ἄλογόν ἐστιν, καὶ ἡ δυναμένη αὐτὸ ἄλογός ἐστιν, καλεῖται δὲ ἐλάττων. δύναται δὲ τὸ ὑπὸ τῶν ΘΒΜ ἡ ΑΒ διὰ τὸ ἐπιζευγνυμένης τῆς ΑΘ ἰσογώνιον γίνεσθαι τὸ ΑΒΘ τρίγωνον τῷ ΑΒΜ τριγώνῳ καὶ εἶναι ὡς τὴν ΘΒ πρὸς τὴν ΒΑ, οὕτως τὴν ΑΒ πρὸς τὴν ΒΜ.

Ἡ ἄρα ΑΒ τοῦ πενταγώνου πλευρὰ ἄλογός ἐστιν ἡ καλουμένη ἐλάττων· ὅπερ ἔδει δεῖξαι.

whole, then the remainder is that irrational (straight-line called) an apotome [Prop. 10.73]. Thus, MB is an apotome, and MK its attachment. So, I say that (it is) also a fourth (apotome). So, let the (square) on N be (made) equal to that (magnitude) by which the (square) on BK is greater than the (square) on KM. Thus, the square on BK is greater than the (square) on KM by the (square) on N. And since KF is commensurable (in length) with FB then, via composition, KB is also commensurable (in length) with FB [Prop. 10.15]. But, BF is commensurable (in length) with BH. Thus, BK is also commensurable (in length) with BH [Prop. 10.12]. And since the (square) on BK is five times the (square) on KM, the (square) on BK thus has to the (square) on KM the ratio which 5 (has) to 1. Thus, via conversion, the (square) on BK has to the (square) on N the ratio which 5 (has) to 4 [Prop. 5.19 corr.], which is not (that) of a square (number) to a square (number). BK is thus incommensurable (in length) with N [Prop. 10.9]. Thus, the square on BK is greater than the (square) on KM by the (square) on (some straight-line which is) incommensurable (in length) with (BA). Therefore, since the square on the whole, BK, is greater than the (square) on the attachment, KM, by the (square) on (some straight-line which is) incommensurable (in length) with (BA), and the whole, BK, is commensurable (in length) with the (previously) laid down rational (straight-line) BH, MB is thus a fourth apotome [Def. 10.14]. And the rectangle contained by a rational (straight-line) and a fourth apotome is irrational, and its square-root is that irrational (straight-line) called minor [Prop. 10.94]. And the square on AB is the rectangle contained by HBM, on account of joining AH, (so that) triangle ABH becomes equiangular with triangle ABM [Prop. 6.8], and (proportionally) as HB is to BA, so AB (is) to BM.

Thus, the side AB of the pentagon is that irrational (straight-line) called minor.† (Which is) the very thing it was required to show.

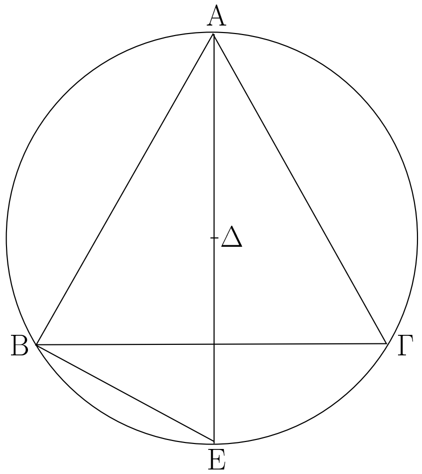
† If the circle has unit radius, then the side of the pentagon is $(1/2)\sqrt{10-2\sqrt{5}}$. However, this length can be written in the "minor" form (see Prop. 10.94) $(\rho/\sqrt{2})\sqrt{1+k/\sqrt{1+k^2}}-(\rho/\sqrt{2})\sqrt{1-k/\sqrt{1+k^2}}$, with $\rho=\sqrt{5/2}$ and $k=2$.

ιβ΄.

Ἐὰν εἰς κύκλον τρίγωνον ἰσόπλευρον ἐγγραφῇ, ἡ τοῦ τριγώνου πλευρὰ δυνάμει τριπλασίων ἐστὶ τῆς ἐκ τοῦ κέντρου τοῦ κύκλου.

Proposition 12

If an equilateral triangle is inscribed in a circle then the square on the side of triangle ABC is three times the (square) on the radius of the circle.

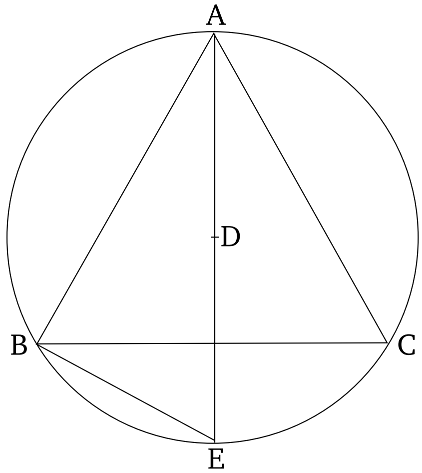


Ἔστω κύκλος ὁ ΑΒΓ, καὶ εἰς αὐτὸν τρίγωνον ἰσόπλευρον ἐγγεγράφθω τὸ ΑΒΓ· λέγω, ὅτι τοῦ ΑΒΓ τριγώνου μία πλευρὰ δυνάμει τριπλασίων ἐστὶ τῆς ἐκ τοῦ κέντρου τοῦ ΑΒΓ κύκλου.

Εἰλήφθω γὰρ τὸ κέντρον τοῦ ΑΒΓ κύκλου τὸ Δ, καὶ ἐπιζευχθεῖσα ἡ ΑΔ διήχθω ἐπὶ τὸ Ε, καὶ ἐπεζεύχθω ἡ ΒΕ.

Καὶ ἐπεὶ ἰσόπλευρόν ἐστι τὸ ΑΒΓ τρίγωνον, ἡ ΒΕΓ ἄρα περιφέρεια τρίτον μέρος ἐστὶ τῆς τοῦ ΑΒΓ κύκλου περιφερείας. ἡ ἄρα ΒΕ περιφέρεια ἕκτον ἐστὶ μέρος τῆς τοῦ κύκλου περιφερείας· ἑξαγώνου ἄρα ἐστὶν ἡ ΒΕ εὐθεῖα· ἴση ἄρα ἐστὶ τῇ ἐκ τοῦ κέντρου τῇ ΔΕ. καὶ ἐπεὶ διπλῆ ἐστιν ἡ ΑΕ τῆς ΔΕ, τετραπλάσιον ἐστι τὸ ἀπὸ τῆς ΑΕ τοῦ ἀπὸ τῆς ΕΔ, τουτέστι τοῦ ἀπὸ τῆς ΒΕ. ἴσον δὲ τὸ ἀπὸ τῆς ΑΕ τοῖς ἀπὸ τῶν ΑΒ, ΒΕ· τὰ ἄρα ἀπὸ τῶν ΑΒ, ΒΕ τετραπλάσιά ἐστι τοῦ ἀπὸ τῆς ΒΕ. διελόντι ἄρα τὸ ἀπὸ τῆς ΑΒ τριπλάσιόν ἐστι τοῦ ἀπὸ ΒΕ. ἴση δὲ ἡ ΒΕ τῇ ΔΕ· τὸ ἄρα ἀπὸ τῆς ΑΒ τριπλάσιόν ἐστι τοῦ ἀπὸ τῆς ΔΕ.

Ἡ ἄρα τοῦ τριγώνου πλευρὰ δυνάμει τριπλασία ἐστὶ τῆς ἐκ τοῦ κέντρου [τοῦ κύκλου]· ὅπερ ἔδει δεῖξαι.



Let there be a circle ABC, and let the equilateral triangle ABC have been inscribed in it [Prop. 4.2]. I say that the square on one side of triangle ABC is three times the (square) on the radius of circle ABC.

For let the center, D, of circle ABC have been found [Prop. 3.1]. And AD (being) joined, let it have been drawn across to E. And let BE have been joined.

And since triangle ABC is equilateral, circumference BEC is thus the third part of the circumference of circle ABC. Thus, circumference BE is the sixth part of the circumference of the circle. Thus, straight-line BE is (the side) of a hexagon. Thus, it is equal to the radius DE [Prop. 4.15 corr.]. And since AE is double DE, the (square) on AE is four times the (square) on ED—that is to say, of the (square) on BE. And the (square) on AE (is) equal to the (sum of the squares) on AB and BE [Props. 3.31, 1.47]. Thus, the (sum of the squares) on AB and BE is four times the (square) on BE. Thus, via separation, the (square) on AB is three times the (square) on BE. And BE (is) equal to DE. Thus, the (square) on AB is three times the (square) on DE.

Thus, the square on the side of the triangle is three times the (square) on the radius [of the circle]. (Which is) the very thing it was required to show.

ιγʹ.

Πυραμίδα συστήσασθαι καὶ σφαίρᾳ περιλαβεῖν τῇ δοθείσῃ καὶ δεῖξαι, ὅτι ἡ τῆς σφαίρας διάμετρος δυνάμει ἡμιολία ἐστὶ τῆς πλευρᾶς τῆς πυραμίδος.

Proposition 13

To construct a (regular) pyramid (*i.e.*, a tetrahedron), and to enclose (it) in a given sphere, and to show that the square on the diameter of the sphere is one and a half times the (square) on the side of the pyramid.