Ὅροι.

α΄. Μέρος ἐστὶ μέγεθος μεγέθους τὸ ἔλασσον τοῦ μείζονος, ὅταν καταμετρῇ τὸ μεῖζον.

β΄. Πολλαπλάσιον δὲ τὸ μεῖζον τοῦ ἐλάττονος, ὅταν καταμετρῆται ὑπὸ τοῦ ἐλάττονος.

γ΄. Λόγος ἐστὶ δύο μεγεθῶν ὁμογενῶν ἡ κατὰ πηλικότητά ποια σχέσις.

δ΄. Λόγον ἔχειν πρὸς ἄλληλα μεγέθη λέγεται, ἃ δύναται πολλαπλασιαζόμενα ἀλλήλων ὑπερέχειν.

ε΄. Ἐν τῷ αὐτῷ λόγῳ μεγέθη λέγεται εἶναι πρῶτον πρὸς δεύτερον καὶ τρίτον πρὸς τέταρτον, ὅταν τὰ τοῦ πρώτου καί τρίτου ἰσάκις πολλαπλάσια τῶν τοῦ δευτέρου καὶ τετάρτου ἰσάκις πολλαπλασίων καθ᾽ ὁποιονοῦν πολλαπλασιασμὸν ἑκάτερον ἑκατέρου ἢ ἅμα ὑπερέχῃ ἢ ἅμα ἴσα ᾖ ἢ ἅμα ἐλλείπῃ ληφθέντα κατάλληλα.

ϛ΄. Τὰ δὲ τὸν αὐτὸν ἔχοντα λόγον μεγέθη ἀνάλογον καλείσθω.

ζ΄. Ὅταν δὲ τῶν ἰσάκις πολλαπλασίων τὸ μὲν τοῦ πρώτου πολλαπλάσιον ὑπερέχῃ τοῦ τοῦ δευτέρου πολλαπλασίου, τὸ δὲ τοῦ τρίτου πολλαπλάσιον μὴ ὑπερέχῃ τοῦ τοῦ τετάρτου πολλαπλασίου, τότε τὸ πρῶτον πρὸς τὸ δεύτερον μείζονα λόγον ἔχειν λέγεται, ἤπερ τὸ τρίτον πρὸς τὸ τέταρτον.

η΄. Ἀναλογία δὲ ἐν τρισὶν ὅροις ἐλαχίστη ἐστίν.

θ΄. Ὅταν δὲ τρία μεγέθη ἀνάλογον ᾖ, τὸ πρῶτον πρὸς τὸ τρίτον διπλασίονα λόγον ἔχειν λέγεται ἤπερ πρὸς τὸ δεύτερον.

ι΄. Ὅταν δὲ τέσσαρα μεγέθη ἀνάλογον ᾖ, τὸ πρῶτον πρὸς τὸ τέταρτον τριπλασίονα λόγον ἔχειν λέγεται ἤπερ πρὸς τὸ δεύτερον, καὶ ἀεὶ ἑξῆς ὁμοίως, ὡς ἂν ἡ ἀναλογία ὑπάρχῃ.

ια΄. Ὁμόλογα μεγέθη λέγεται τὰ μὲν ἡγούμενα τοῖς ἡγουμένοις τὰ δὲ ἑπόμενα τοῖς ἑπομένοις.

ιβ΄. Ἐναλλὰξ λόγος ἐστὶ λῆψις τοῦ ἡγουμένου πρὸς τὸ ἡγούμενον καὶ τοῦ ἑπομένου πρὸς τὸ ἑπόμενον.

ιγ΄. Ἀνάπαλιν λόγος ἐστὶ λῆψις τοῦ ἑπομένου ὡς ἡγουμένου πρὸς τὸ ἡγούμενον ὡς ἑπόμενον.

ιδ΄. Σύνθεσις λόγου ἐστὶ λῆψις τοῦ ἡγουμένου μετὰ τοῦ ἑπομένου ὡς ἑνὸς πρὸς αὐτὸ τὸ ἑπόμενον.

ιε΄. Διαίρεσις λόγου ἐστὶ λῆψις τῆς ὑπεροχῆς, ᾗ ὑπερέχει τὸ ἡγούμενον τοῦ ἑπομένου, πρὸς αὐτὸ τὸ ἑπόμενον.

ιϛ΄. Ἀναστροφὴ λόγου ἐστὶ λῆψις τοῦ ἡγουμένου πρὸς τὴν ὑπεροχήν, ᾗ ὑπερέχει τὸ ἡγούμενον τοῦ ἑπομένου.

ιζ΄. Δι᾽ ἴσου λόγος ἐστὶ πλειόνων ὄντων μεγεθῶν καὶ ἄλλων αὐτοῖς ἴσων τὸ πλῆθος σύνδυο λαμβανομένων καὶ ἐν τῷ αὐτῷ λόγῳ, ὅταν ᾖ ὡς ἐν τοῖς πρώτοις μεγέθεσι

Definitions

1. A magnitude is a part of a(nother) magnitude, the lesser of the greater, when it measures the greater.†

2. And the greater (magnitude is) a multiple of the lesser when it is measured by the lesser.

3. A ratio is a certain type of condition with respect to size of two magnitudes of the same kind.‡

4. (Those) magnitudes are said to have a ratio with respect to one another which, being multiplied, are capable of exceeding one another.§

5. Magnitudes are said to be in the same ratio, the first to the second, and the third to the fourth, when equal multiples of the first and the third either both exceed, are both equal to, or are both less than, equal multiples of the second and the fourth, respectively, being taken in corresponding order, according to any kind of multiplication whatever.¶

6. And let magnitudes having the same ratio be called proportional.*

7. And when for equal multiples (as in Def. 5), the multiple of the first (magnitude) exceeds the multiple of the second, and the multiple of the third (magnitude) does not exceed the multiple of the fourth, then the first (magnitude) is said to have a greater ratio to the second than the third (magnitude has) to the fourth.

8. And a proportion in three terms is the smallest (possible).$

9. And when three magnitudes are proportional, the first is said to have a squared‖ ratio to the third with respect to the second.††

10. And when four magnitudes are (continuously) proportional, the first is said to have a cubed‡‡ ratio to the fourth with respect to the second.§§ And so on, similarly, in successive order, whatever the (continuous) proportion might be.

11. These magnitudes are said to be corresponding (magnitudes): the leading to the leading (of two ratios), and the following to the following.

12. An alternate ratio is a taking of the (ratio of the) leading (magnitude) to the leading (of two equal ratios), and (setting it equal to) the (ratio of the) following (magnitude) to the following.¶¶

13. An inverse ratio is a taking of the (ratio of the) following (magnitude) as the leading and the leading (magnitude) as the following.**

14. A composition of a ratio is a taking of the (ratio of the) leading plus the following (magnitudes), as one, to the same following (magnitude).$$

τὸ πρῶτον πρὸς τὸ ἔσχατον, οὕτως ἐν τοῖς δευτέροις μεγέθεσι τὸ πρῶτον πρὸς τὸ ἔσχατον· ἢ ἄλλως· λῆψις τῶν ἄκρων καθ᾽ ὑπεξαίρεσιν τῶν μέσων.

ιη΄. Τεταραγμένη δὲ ἀναλογία ἐστίν, ὅταν τριῶν ὄντων μεγεθῶν καὶ ἄλλων αὐτοῖς ἴσων τὸ πλῆθος γίνηται ὡς μὲν ἐν τοῖς πρώτοις μεγέθεσιν ἡγούμενον πρὸς ἑπόμενον, οὕτως ἐν τοῖς δευτέροις μεγέθεσιν ἡγούμενον πρὸς ἑπόμενον, ὡς δὲ ἐν τοῖς πρώτοις μεγέθεσιν ἑπόμενον πρὸς ἄλλο τι, οὕτως ἐν τοῖς δευτέροις ἄλλο τι πρὸς ἡγούμενον.

15. A separation of a ratio is a taking of the (ratio of the) excess by which the leading (magnitude) exceeds the following to the same following (magnitude).‖‖

16. A conversion of a ratio is a taking of the (ratio of the) leading (magnitude) to the excess by which the leading (magnitude) exceeds the following.†††

17. There being several magnitudes, and other (magnitudes) of equal number to them, (which are) also in the same ratio taken two by two, a ratio via equality (or *ex aequali*) occurs when as the first is to the last in the first (set of) magnitudes, so the first (is) to the last in the second (set of) magnitudes. Or alternately, (it is) a taking of the (ratio of the) outer (magnitudes) by the removal of the inner (magnitudes).‡‡‡

18. There being three magnitudes, and other (magnitudes) of equal number to them, a perturbed proportion occurs when as the leading is to the following in the first (set of) magnitudes, so the leading (is) to the following in the second (set of) magnitudes, and as the following (is) to some other (*i.e.*, the remaining magnitude) in the first (set of) magnitudes, so some other (is) to the leading in the second (set of) magnitudes.§§§

† In other words, α is said to be a part of β if $\beta = m\,\alpha$.

‡ In modern notation, the ratio of two magnitudes, α and β, is denoted $\alpha : \beta$.

§ In other words, α has a ratio with respect to β if $m\,\alpha > \beta$ and $n\,\beta > \alpha$, for some m and n.

¶ In other words, $\alpha : \beta :: \gamma : \delta$ if and only if $m\,\alpha > n\,\beta$ whenever $m\,\gamma > n\,\delta$, and $m\,\alpha = n\,\beta$ whenever $m\,\gamma = n\,\delta$, and $m\,\alpha < n\,\beta$ whenever $m\,\gamma < n\,\delta$, for all m and n. This definition is the kernel of Eudoxus' theory of proportion, and is valid even if α, β, *etc.*, are irrational.

* Thus if α and β have the same ratio as γ and δ then they are proportional. In modern notation, $\alpha : \beta :: \gamma : \delta$.

$ In modern notation, a proportion in three terms—α, β, and γ—is written: $\alpha : \beta :: \beta : \gamma$.

‖ Literally, "double".

†† In other words, if $\alpha : \beta :: \beta : \gamma$ then $\alpha : \gamma :: \alpha^2 : \beta^2$.

‡‡ Literally, "triple".

§§ In other words, if $\alpha : \beta :: \beta : \gamma :: \gamma : \delta$ then $\alpha : \delta :: \alpha^3 : \beta^3$.

¶¶ In other words, if $\alpha : \beta :: \gamma : \delta$ then the alternate ratio corresponds to $\alpha : \gamma :: \beta : \delta$.

** In other words, if $\alpha : \beta$ then the inverse ratio corresponds to $\beta : \alpha$.

$$ In other words, if $\alpha : \beta$ then the composed ratio corresponds to $\alpha + \beta : \beta$.

‖‖ In other words, if $\alpha : \beta$ then the separated ratio corresponds to $\alpha - \beta : \beta$.

††† In other words, if $\alpha : \beta$ then the converted ratio corresponds to $\alpha : \alpha - \beta$.

‡‡‡ In other words, if α, β, γ are the first set of magnitudes, and δ, ϵ, ζ the second set, and $\alpha : \beta : \gamma :: \delta : \epsilon : \zeta$, then the ratio via equality (or *ex aequali*) corresponds to $\alpha : \gamma :: \delta : \zeta$.

§§§ In other words, if α, β, γ are the first set of magnitudes, and δ, ϵ, ζ the second set, and $\alpha : \beta :: \delta : \epsilon$ as well as $\beta : \gamma :: \zeta : \delta$, then the proportion is said to be perturbed.

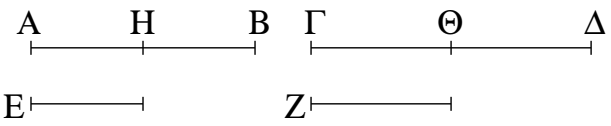
α΄.

᾽Εὰν ᾖ ὁποσαοῦν μεγέθη ὁποσωνοῦν μεγεθῶν ἴσων τὸ πλῆθος ἕκαστον ἑκάστου ἰσάκις πολλαπλάσιον,

Proposition 1†

If there are any number of magnitudes whatsoever (which are) equal multiples, respectively, of some (other)

ὁσαπλάσιόν ἐστιν ἓν τῶν μεγεθῶν ἑνός, τοσαυταπλάσια ἔσται καὶ τὰ πάντα τῶν πάντων.

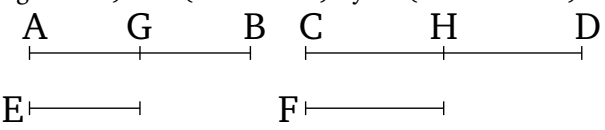


Ἔστω ὁποσαοῦν μεγέθη τὰ ΑΒ, ΓΔ ὁποσωνοῦν μεγεθῶν τῶν Ε, Ζ ἴσων τὸ πλῆθος ἕκαστον ἑκάστου ἰσάκις πολλαπλάσιον· λέγω, ὅτι ὁσαπλάσιόν ἐστι τὸ ΑΒ τοῦ Ε, τοσαυταπλάσια ἔσται καὶ τὰ ΑΒ, ΓΔ τῶν Ε, Ζ.

Ἐπεὶ γὰρ ἰσάκις ἐστὶ πολλαπλάσιον τὸ ΑΒ τοῦ Ε καὶ τὸ ΓΔ τοῦ Ζ, ὅσα ἄρα ἐστὶν ἐν τῷ ΑΒ μεγέθη ἴσα τῷ Ε, τοσαῦτα καὶ ἐν τῷ ΓΔ ἴσα τῷ Ζ. διῃρήσθω τὸ μὲν ΑΒ εἰς τὰ τῷ Ε μεγέθη ἴσα τὰ ΑΗ, ΗΒ, τὸ δὲ ΓΔ εἰς τὰ τῷ Ζ ἴσα τὰ ΓΘ, ΘΔ· ἔσται δὴ ἴσον τὸ πλῆθος τῶν ΑΗ, ΗΒ τῷ πλήθει τῶν ΓΘ, ΘΔ. καὶ ἐπεὶ ἴσον ἐστὶ τὸ μὲν ΑΗ τῷ Ε, τὸ δὲ ΓΘ τῷ Ζ, ἴσον ἄρα τὸ ΑΗ τῷ Ε, καὶ τὰ ΑΗ, ΓΘ τοῖς Ε, Ζ. διὰ τὰ αὐτὰ δὴ ἴσον ἐστὶ τὸ ΗΒ τῷ Ε, καὶ τὰ ΗΒ, ΘΔ τοῖς Ε, Ζ· ὅσα ἄρα ἐστὶν ἐν τῷ ΑΒ ἴσα τῷ Ε, τοσαῦτα καὶ ἐν τοῖς ΑΒ, ΓΔ ἴσα τοῖς Ε, Ζ· ὁσαπλάσιον ἄρα ἐστὶ τὸ ΑΒ τοῦ Ε, τοσαυταπλάσια ἔσται καὶ τὰ ΑΒ, ΓΔ τῶν Ε, Ζ.

Ἐὰν ἄρα ᾖ ὁποσαοῦν μεγέθη ὁποσωνοῦν μεγεθῶν ἴσων τὸ πλῆθος ἕκαστον ἑκάστου ἰσάκις πολλαπλάσιον, ὁσαπλάσιόν ἐστιν ἓν τῶν μεγεθῶν ἑνός, τοσαυταπλάσια ἔσται καὶ τὰ πάντα τῶν πάντων· ὅπερ ἔδει δεῖξαι.

magnitudes, of equal number (to them), then as many times as one of the (first) magnitudes is (divisible) by one (of the second), so many times will all (of the first magnitudes) also (be divisible) by all (of the second).



Let there be any number of magnitudes whatsoever, AB, CD, (which are) equal multiples, respectively, of some (other) magnitudes, E, F, of equal number (to them). I say that as many times as AB is (divisible) by E, so many times will AB, CD also be (divisible) by E, F.

For since AB, CD are equal multiples of E, F, thus as many magnitudes as (there) are in AB equal to E, so many (are there) also in CD equal to F. Let AB have been divided into magnitudes AG, GB, equal to E, and CD into (magnitudes) CH, HD, equal to F. So, the number of (divisions) AG, GB will be equal to the number of (divisions) CH, HD. And since AG is equal to E, and CH to F, AG (is) thus equal to E, and AG, CH to E, F. So, for the same (reasons), GB is equal to E, and GB, HD to E, F. Thus, as many (magnitudes) as (there) are in AB equal to E, so many (are there) also in AB, CD equal to E, F. Thus, as many times as AB is (divisible) by E, so many times will AB, CD also be (divisible) by E, F.

Thus, if there are any number of magnitudes whatsoever (which are) equal multiples, respectively, of some (other) magnitudes, of equal number (to them), then as many times as one of the (first) magnitudes is (divisible) by one (of the second), so many times will all (of the first magnitudes) also (be divisible) by all (of the second). (Which is) the very thing it was required to show.

† In modern notation, this proposition reads $m\,\alpha + m\,\beta + \cdots = m\,(\alpha + \beta + \cdots)$.

β΄.

Ἐὰν πρῶτον δευτέρου ἰσάκις ᾖ πολλαπλάσιον καὶ τρίτον τετάρτου, ᾖ δὲ καὶ πέμπτον δευτέρου ἰσάκις πολλαπλάσιον καὶ ἕκτον τετάρτου, καὶ συντεθὲν πρῶτον καὶ πέμπτον δευτέρου ἰσάκις ἔσται πολλαπλάσιον καὶ τρίτον καὶ ἕκτον τετάρτου.

Πρῶτον γὰρ τὸ ΑΒ δευτέρου τοῦ Γ ἰσάκις ἔστω πολλαπλάσιον καὶ τρίτον τὸ ΔΕ τετάρτου τοῦ Ζ, ἔστω δὲ καὶ πέμπτον τὸ ΒΗ δευτέρου τοῦ Γ ἰσάκις πολλαπλάσιον καὶ ἕκτον τὸ ΕΘ τετάρτου τοῦ Ζ· λέγω, ὅτι καὶ συντεθὲν πρῶτον καὶ πέμπτον τὸ ΑΗ δευτέρου τοῦ Γ ἰσάκις ἔσται πολλαπλάσιον καὶ τρίτον καὶ ἕκτον τὸ ΔΘ τετάρτου τοῦ Ζ.

Proposition 2†

If a first (magnitude) and a third are equal multiples of a second and a fourth (respectively), and a fifth (magnitude) and a sixth (are) also equal multiples of the second and fourth (respectively), then the first (magnitude) and the fifth, being added together, and the third and the sixth, (being added together), will also be equal multiples of the second (magnitude) and the fourth (respectively).

For let a first (magnitude) AB and a third DE be equal multiples of a second C and a fourth F (respectively). And let a fifth (magnitude) BG and a sixth EH also be (other) equal multiples of the second C and the fourth F (respectively). I say that the first (magnitude) and the fifth, being added together, (to give) AG, and the