Mathematics-Science 3810H – Ancient and classical mathematics
Trent University, Winter 2016

Take-home Final Examination
Due on Monday, 18 April, 2016.

Instructions: Give complete answers to receive full credit, including references to any and all sources you used. You may use your texts from this and any other courses, as well as any handouts, class notes, and the like; you may also ask the instructor to clarify the instructions or any of the questions; and you may use a calculator or computer to perform any necessary calculations. *You may not consult any other sources, nor give or receive any other aid on this exam, except with the intructor's explicit permission or as otherwise indicated on a given problem.*

Part I – This, and that, and something else. Do all three of **1 – 3**.

1. Answer all of **a – i**. *[10 = 10×1 each]*

    a. Who is the earliest mathematician for whom we have a name?

    b. What was the first real success in mathematical physics?

    c. Name three Greek or Hellenistic mathematicians who also worked on astronomy.

    d. What is the earliest surviving record of numbers?

    e. Which mathematician was reputed to be the second-best at everything in their own lifetime?

    f. According to Herodotus, why did the ancient Egyptians invent geometry?

    g. Which of the number systems mentioned in this course required the fewest distinct symbols and which required the most?

    h. Name an example of an ancient work on mathematics that used results from earlier works without attribution.

    i. Name as many areas of Greek or Hellenistic mathematics as you can that were at least somewhat developed in Euclid's time, but did not make it into his *Elements*.

    j. Who classified astronomy as a branch of mathematics?

2. Did the early development of geometry and the limitations of the number systems used in classical times inhibit the development of algebra and algebraic notation? *[15]*

3. A *median* of a triangle connects a vertex to the midpoint of the opposite side. The three medians of any triangle are concurrent in a point (the *centroid* of the triangle). Find out whether this result was known in Greek or Hellenistic times. If it was, who did it first and what was their proof? If not, who did get there first and why did no one in Greek or Hellenistic times do so? [*You may consult additional sources for this problem.*] *[15]*

Part II – History. Do *one* of **4** and **5**. You may use additional sources for either one.

4. How did the development of mathematics affect the development of physics and astronomy in ancient and classical times? *[15]*

5. Compare and contrast the number systems used in ancient Mesopotamia and Egypt. What are the advantages and disadvantages of each? *[15]*

Part III – Mathematics. Do any *two* of **6** – **8**. *[20 = 2 ×10 each]*

6. Describe one of the methods developed by Greek or Hellenistic mathematicians to trisect an angle with more than just straightedge and compass and explain in detail why the method works. *[10]*

7. If u and v are two numbers and $u \geq v$, their *average* is $(u+v)/2$ and their *semidifference* is $(u - v)/2$. Do all three of **a** – **c**.
   a. Express uv in terms of the average and semidifference of u and v. *[3]*
   b. Given that $a = u + v$ and $b = uv$, solve for u and v in terms of a and b. *[3]*
   c. How can the method in **b** be used to solve quadratic equations? *[4]*

8. Do both of **a** and **b**.
   a. Suppose n and $2^n - 1$ are both prime. Show that $2^{n-1}(2^n - 1)$ is a perfect number. *[7]*
   b. Would $2^{n-1}(2^n - 1)$ still have to be a perfect number if n were not prime? Why or why not? *[3]*

[Total = 75]

Part IV - Bonus. For fun *and* ~~profit~~ marks!

◎. A *snowball sentence* is a sentence in which each word has one more letter than the preceding word. (*e.g.* "I am not dumb; smart person present!") Write as long a snowball sentence as you can touching on the history of mathematics. *[1]*

I HOPE THAT YOU HAD SOME FUN WITH THIS.
ENJOY THE SUMMER!