

Mathematics 3810H – Ancient and classical mathematics

TRENT UNIVERSITY, Winter 2016

Assignment #3

Magnitudes

Due on Thursday, 25 February, 2016.

Eudoxus of Cnidus (c. 410–350 B.C.) was one of the great early Greek mathematicians, who also worked in astronomy and philosophy. Unfortunately, all of his writings are now lost, and we only have the word of (sometimes much) later commentators for what he accomplished. The theory of proportions Eudoxus is supposed to have developed allowed ancient Greek mathematicians to rigorously handle quantities that were not necessarily rational. Some of it seems very strange to modern eyes and ears, though. For one thing, the quantities in question were conceived of as geometric magnitudes, such as length or area, rather than numbers in a number system extending the rationals.

Consider the following two statements, taken from a translation of Euclid's *Elements*, the relevant part of which is believed to be based on Eudoxus' work on proportions:

DEFINITION. Magnitudes are said to be in the same ratio, the first to the second, and the third to the fourth, when equal multiples of the first and the third either both exceed, are both equal to, or are both less than, equal multiples of the second and the fourth, respectively, being taken in corresponding order, according to any kind of multiplication whatever.

PROPOSITION. If there are any number of magnitudes whatsoever which are equal multiples, respectively, of some other magnitudes, of equal number to them, then as many times as one of the first magnitudes is divisible by one of the second, so many times will all of the first magnitudes also be divisible by all of the second.

1. What does the definition given above state, in modern terms? [2]
2. What does the proposition given above state, in modern terms? [2]
3. What is the significance of the definition and proposition given above? [2]
4. Try to prove the given proposition both in modern terms and in the style in which it is stated. [4]

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Euclid alone has looked on Beauty bare.
Let all who prate of Beauty hold their peace,
And lay them prone upon the earth and cease
To ponder on themselves, the while they stare
At nothing, intricately drawn nowhere
In shapes of shifting lineage; let geese
Gabble and hiss, but heroes seek release
From dusty bondage into luminous air.
O blinding hour, O holy, terrible day,
When first the shaft into his vision shone
Of light anatomized! Euclid alone
Has looked on Beauty bare. Fortunate they
Who, though once only and then but far away,
Have heard her massive sandal set on stone.

Edna St. Vincent Millay