Mathematics 3810H – Ancient and classical mathematics
Trent University, Fall 2017

Take-home Final Examination
Due on ~~Friday~~ Saturday, 16 December, 2017.

Instructions: Do all three of Parts $\boldsymbol{\alpha}$ – $\boldsymbol{\gamma}$ and, if you wish, Part $\boldsymbol{\delta}$ as well. Give complete answers to receive full credit, including references to any and all sources you used. You may use your texts from this and any other courses, as well as any handouts, class notes, and the like; you may also ask the instructor to clarify the instructions or any of the questions; and you may use a calculator or computer to perform any necessary calculations. *You may not consult any other sources, nor give or receive any other aid on this exam, except with the intructor's explicit permission or as otherwise indicated for a given problem.*

Part $\boldsymbol{\alpha}$ – This, that, and the other thing. Do all three of **1** – **3**.

1. Answer all of **a** – **i**. *[10 = 10×1 each]*

- **a.** Is there any evidence that the ancient Mesopotamians ever did mathematics for its own sake? If so, what is it?
- **b.** Who first systematically used a symbolic notation for algebraic expressions?
- **c.** Which ancient mathematician was supposedly second-best at everything?
- **d.** Why is Euclid's Third Postulate necessary?
- **e.** What are the earliest written numbers?
- **f.** Why were reciprocals so important in Mesopotamian mathematics?
- **g.** Name three ancient mathematicians who worked on conic sections.
- **h.** Who first found a connection between mathematics and music?
- **i.** What are the five Platonic solids?
- **j.** What is the only known problem where the Egyptians used the method of single false position to solve a non-linear equation?

2. Compare and contrast the relative strengths and weaknesses of the number systems and associated arithmetical techniques used in ancient Egypt and Mesopotamia. *[15]*

3. Let $|PQ|$ denote the length of the line segment joining the points P and Q. Prove Ptolemy's Theorem ...

> *Suppose A, B, C, and D are any four points on a circle listed in clockwise order. Then $|AB| \cdot |CD| + |AD| \cdot |BC| = |AC| \cdot |BD|$.*

... and show that the equation need not be true if the quadrilateral $ABCD$ is not inscribed in a circle. *[10]*

[Parts $\boldsymbol{\beta}$ – $\boldsymbol{\delta}$ are on page 2.]

Part β – History. Do *one* of **4** and **5**. *[You may consult any sources you like for these.]*

4. Describe Plato's influence on the development of Greek and Hellenistic mathematics. What were its benefits and drawbacks? *[15]*

5. How might Greek and Hellenistic astronomy and mathematics have evolved without the influence of Mesopotamian astronomy and mathematics? *[15]*

Part γ – Mathematics. Do any *two* of **6** – **8**.

6. Describe Plato's solution to the problem of the duplication of the cube, as given in §4.5 of the text, and explain why it works. *[10]*

7. If u and v are two numbers and $u \geq v$, their *average* is $(u+v)/2$ and their *semidifference* is $(u-v)/2$. Do all three of **a** – **c**.

a. Express uv in terms of the average and semidifference of u and v. *[4]*

b. Given that $a = u + v$ and $b = uv$, solve for u and v in terms of a and b. *[2]*

c. How can the method in **b** be used to solve quadratic equations? *[4]*

8. Starting with a line segment of length 1, give a detailed ruler and compass construction of a line segment of length $\sqrt{5}$. *[You may consult any sources you like for this.]* *[10]*

[Total = 70]

Part δ. *Bonus!*

□△. Write a poem touching on mathematics or its history. *[1]*

▲■. One of our few sources for the life of Diophantus is a puzzle given in an anthology by a Fifth Century A.D. writer named Metrodorus:

> 'Here lies Diophantus,' the wonder behold.
> Through art algebraic, the stone tells how old:
> 'God gave him his boyhood one-sixth of his life,
> One twelfth more as youth while whiskers grew rife;
> And then yet one-seventh ere marriage begun;
> In five years there came a bouncing new son.
> Alas, the dear child of master and sage
> After attaining half the measure of his father's life
> chill fate took him.
> After consoling his fate by the science of numbers
> for four years, he ended his life.'

According to the riddle, how old was Diophantus when he died? *[1]*

I HOPE THAT YOU ENJOYED THE COURSE.
HAVE A GOOD BREAK!