Mathematics 3810H – Ancient and classical mathematics
Trent University, Fall 2017

Assignment #5
Modern Ancient Astronomy
Due on Friday, 24 November, 2017.

The paragraph below, from the description of *A Modern Almagest: An updated version of Ptolemy's Almagest*, by Richard Fitzpatrick, at the book's web page at

`http://farside.ph.utexas.edu/books/Syntaxis/Syntaxis.html`

(from which the book is freely dowloadable), gives a good summary of the main reasons why the Ptolemaic model of the solar system was such a successful scientific theory.

> The *Almagest* of Claudius Ptolemy is often unfairly disparaged by modern commentators for a number of supposed failings. Firstly, it is often alleged that Ptolemy's adoption of circular orbits is a major source of error in his model. This is untrue. Planetary orbits are highly circular, due to their relatively small eccentricities (the departure from circularity scales as the eccentricity, e, squared). It is far more important to get the eccentricities of these orbits (i.e., the displacement of the Sun from the geometric center, which scales as e) correct, rather than the ellipticities. This is exactly what Ptolemy does in the *Almagest*. Secondly, it is generally supposed that Ptolemy's introduction of epicycles into his model is merely a clumsy way of retaining a geocentric model whilst allowing for retrograde motion. This is untrue. The deferents and epicycles in the *Almagest* have a clear physical significance. For a superior planet, the deferent represents the orbit of the planet around the Sun, whereas the epicycle represents the Earth's orbit. The opposite is true for an inferior planet. Thirdly, it is often alleged that Ptolemy's model requires epicycles on epicycles, and epicycles on epicycles on epicycles, etc., in order to fit the observational data to any degree of accuracy. This is untrue. In the first place, Ptolemy never proposed more than one level of epicycles. In the second place, a (slightly corrected) version of Ptolemy's model containing only 10 circles (four deferents and five epicycles) is capable of accounting for the apparent movements of the Sun and the five visible planets to an accuracy which is more than adequate for naked-eye astronomy. It is worth noting that Ptolemy's model contains fewer epicycles than that proposed by Copernicus, and is also more accurate.

One interesting point not mentioned in this paragraph is that epicycles can actually generate ellipses. Suppose, for example, that we start with the circle of radius 1 centred at $(2,0)$. Suppose further that we spin this circle clockwise about its centre while moving its centre counterclockwise along the circle $x^2+y^2=4$ at the same constant angular rate. (That is, each circle completes a full revolution about its centre at the same time.) The point which is initially at $(3,0)$ then traces out an ellipse.

1. Verify that the point initially at $(3,0)$ traces out the ellipse $\dfrac{x^2}{9}+y^2=1$. *[6]*

Note: Feel free to use modern mathematical tools, including trigonometry.

2. Can every ellipse be traced out using epicycles? If so, explain how. If not, explain why not and try to figure which ones can be so obtained. *[4]*