Mathematics 3810H – Ancient and classical mathematics
Trent University, Fall 2017

Assignment #3
Due on Friday, 20 October.

Eudoxus of Cnidus (*c.* 410–350 B.C.) was one of the great early Greek mathematicians, who also worked in astronomy and philosophy. Unfortunately, all of his writings are now lost, and we only have the word of (sometimes much) later commentators for what he accomplished. The theory of proportions Eudoxus is supposed to have developed allowed ancient Greek mathematicians to rigourously handle quantities that were not necessarily rational. Some of it seems very strange to modern eyes and ears, though, partly because all the quantities in question were conceived of as geometric magnitudes, such as length or area, rather than numbers in a number system extending the rationals.

One of the things that Eudoxus is supposed to have proved is that the area of a circle is proportional to the square of its diameter. It is likely that proof of this fact in Book XII of Euclid's *Elements* is based on Eudoxus' work. The argument given there is based on inscribing regular polygons with 2^n sides in circles; in this assignment you will work through a variation of this argument.

1. A regular 2^n-gon inscribed in a circle of radius r has area $2^{n-1}r^2\sin\left(\dfrac{\pi}{2^{n-1}}\right)$. *[3]*

Note: Eudoxus did his work before trigonometric functions were known, so he could not have had this result.

2. Prove that a regular 2^n-gon inscribed in a circle takes up more than $1-\dfrac{1}{2^{n-1}}$ of the area of the circle. *[4]*

3. Use the result in **2** above to prove that the area of a circle is proportional to the square of its radius or diameter. *[3]*

On Problems

Our choicest plans
    have fallen through,
our airiest castles
    tumbled over,
because of lines
    we neatly drew
and later neatly
    stumbled over.

Piet Hein