

**Mathematics-Science 3810H – Ancient and classical mathematics**

TRENT UNIVERSITY, Fall 2013

TAKE-HOME FINAL EXAMINATION

*Due on Friday, 20 December, 2013.*

**Instructions:** Give complete answers to receive full credit, including references to any and all sources you used. You may use your texts from this and any other courses, as well as any handouts, class notes, and the like; you may also ask the instructor to clarify the instructions or any of the questions; and you may use a calculator or computer to perform any necessary calculations. *You may not consult any other sources, nor give or receive any other aid on this exam, except with the instructor's explicit permission or as otherwise indicated on a given problem.*

**Part I – This, and that, and something else.** Do all three of **1 – 3**.

1. Answer all of **a – i**. [ $10 = 10 \times 1$  each]
  - a. Which mathematical tradition used the method of false position?
  - b. Why is Euclid's Third Postulate necessary?
  - c. Name an example of an ancient work on mathematics that used results from earlier works without attribution.
  - d. What is the earliest surviving record of numbers?
  - e. Who first systematically used a symbolic notation for algebraic expressions?
  - f. Why were reciprocals so important in Mesopotamian mathematics?
  - g. Which of the number systems studied in this course required the fewest distinct symbols and which required the most?
  - h. Who seems to have been the first to do proofs in mathematics?
  - i. Name three Greek works touching on mathematics that were each originally published in 13 books.
  - j. Who classified music as a branch of mathematics?
2. Compare and contrast the relative strengths and weaknesses of the number systems used in ancient Egypt, Mesopotamia, and Greece. [15]
3. Recall Ptolemy's Theorem from class:

*Suppose  $A, B, C,$  and  $D$  are any four points on a circle listed in clockwise order. Then  $|AB| \cdot |CD| + |AD| \cdot |BC| = |AC| \cdot |BD|$ .*

Use Ptolemy's Theorem to prove the following identity for the chord function (in a unit circle, with angles in radians):

$$\text{chord}(\alpha - \beta) = \text{chord}(\alpha)\text{chord}(\pi - \beta) - \text{chord}(\beta)\text{chord}(\pi - \alpha) \quad [15]$$

*Hint:* Make  $AD$  a diameter in Ptolemy's Theorem and let  $\alpha$  and  $\beta$  be the angles ...

[Parts **II – IV** are on page 2.]

**Part II – History.** Do *one* of 4 and 5. You may use additional sources for either one.

4. How was the development of mathematics affected by the needs of astrology? [15]
5. One thing we did not seriously consider in class were the methods were used in Greek and Hellenistic times to do arithmetic. Find out and describe how they actually performed the basic arithmetic operations. [15]

**Part III – Mathematics.** Do any *two* of 6 – 8.

6. Describe how to use the *quadratrix* to trisect an angle and explain in detail why the method works. [You may consult additional sources for this problem.] [10]
7. Consider the following method for approximating  $\sqrt{2}$ , a description in modern notation of a method used in Mesopotamia:

$$\text{Let } a_0 = 1. \text{ Given } a_n, \text{ let } a_{n+1} = \frac{1}{2} \left( a_n + \frac{2}{a_n} \right).$$

- a. Compute  $a_n$  for  $n = 1, 2, 3$ . How close is each of these to  $\sqrt{2}$ ? [2]
  - b. Show that  $\lim_{n \rightarrow \infty} a_n$  exists and equals  $\sqrt{2}$ . [5]
  - c. Adapt this method to get way to approximate  $\sqrt{7}$ . [3]
8. Given a line segment of length 1 as a reference, give a ruler and compass construction of a line segment of the length of the *golden section*,  $\varphi = \frac{-1+\sqrt{5}}{2}$ . [5]

[Total = 75]

**Part IV - Old school algebra? Bonus!**

$\Delta^v$ . Answer the riddle below, which supposedly gives the length of Diophantus' life. [1]

126.—ΑΛΛΟ

Οὗτός τοι Διόφαντον ἔχει τάφος· ἃ μέγα θαῦμα·  
καὶ τάφος ἐκ τέχνης μέτρα βίοιο λέγει.  
ἕκτην κουρίζειν βίοτου θεὸς ὥπασε μοίρην·  
δωδεκάτην δ' ἐπιθείς, μῆλα πόρεν χροάειν·  
τῇ δ' ἄρ' ἐφ' ἑβδομάτῃ τὸ γαμήλιον ἤψατο φέγγος, ἃ  
ἐκ δὲ γάμων πέμπτῳ παῖδ' ἐπένευσεν ἔτει.  
αἰᾶτ', τηλύγετον δειλὸν τέκος, ἤμισυ πατρός  
ἦτοῦδε καὶ ἡ κρυερὸς μέτρον ἐλὼν βίοτου.  
πένθος δ' αὖ πεισύρεσσι παρηγορέων ἐνιαυτοῖς  
τῆδε πόσου σοφίῃ τέρῳ ἐπέρησε βίου.

10

126

This tomb holds Diophantus. Ah, how great a marvel! the tomb tells scientifically the measure of his life. God granted him to be a boy for the sixth part of his life, and adding a twelfth part to this, he clothed his cheeks with down; He lit him the light of wedlock after a seventh part, and five years after his marriage He granted him a son. Alas! late-born wretched child; after attaining the measure of half his father's life, chill Fate took him. After consoling his grief by this science of numbers for four years he ended his life.

Metrodorus, Epigram 126, Greek Anthology

I HOPE THAT YOU ENJOYED THE COURSE.  
NOW ENJOY THE BREAK!