Mathematics-Science 3810H – Ancient and classical mathematics
Trent University, Fall 2009

Take-home Final Examination
Due on Tuesday, 22 December, 2009.

Instructions: Give complete answers to receive full credit, including references to any and all sources you used. You may use your texts from this and any other courses, as well as any handouts, class notes, and the like; you may also ask the instructor to clarify the instructions or any of the questions; and you may use a calculator or computer to perform any necessary calculations. *You may not consult any other sources, nor give or receive any other aid on this exam, except with the intructor's explicit permission or as otherwise indicated on a given problem.*

Part I – This, that, and the other thing. Do all three of **1** – **3**.

1. Answer all of **a** – **i**. *[10 = 10×1 each]*

- **a.** Is there any evidence that the ancient Mesopotamians ever did mathematics for its own sake? If so, what is it?
- **b.** Who first systematically used a symbolic notation for algebraic expressions?
- **c.** Which ancient mathematician was supposedly second-best at everything?
- **d.** Why is Euclid's Third Postulate necessary?
- **e.** What are the earliest written numbers?
- **f.** Why were reciprocals so important in Mesopotamian mathematics?
- **g.** Name three ancient mathematicians who worked on conic sections.
- **h.** Who first found a connection between mathematics and music?
- **i.** What are the five Platonic solids?
- **j.** What is the only known problem where the Egyptians used the method of single false position to solve a non-linear equation?

2. Compare and contrast the relative strengths and weaknesses of the number systems and associated arithmetical techniques used in ancient Egypt and Mesopotamia. *[15]*

3. The Pythagoreans knew that the sum of the first n odd numbers is n^2. For example, $1 = 1^2$, $1 + 3 = 4 = 2^2$, $1 + 3 + 5 = 9 = 3^2$, and so on. Prove this result. For fullest credit, do it as the Pythagoreans might have done it! *[10]*

Part II – History. Do *one* of **4** and **5**.

4. Describe Plato's influence on the development of Greek and Hellenistic mathematics. What were its benefits and drawbacks? *[15]*

5. How might Greek and Hellenistic astronomy and associated mathematics have evolved without the influence of Mesopotamian astronomy and mathematics? *[15]*

[Parts **III** and **IV** are on page 2.]

Part III – Mathematics. Do any *two* of **6** – **8**.

6. Describe Plato's solution to the problem of the duplication of the cube, as given in §4.5 of the text, and do Exercise 3 of §4.5. *[10]*

7. Consider the following method for approximating $\sqrt{2}$, a description in modern notation of a method used in Mesopotamia:

Let $a_0 = 1$. Given a_n, let $a_{n+1} = \frac{1}{2}\left(a_n + \frac{2}{a_n}\right)$.

a. Compute a_n for $n = 1, 2, 3, 4$. How close is each of these to $\sqrt{2}$? *[2]*

b. What happens if $a_n^2 = 2$ for some n? *[1]*

c. Is a_{n+1} necessarily nearer to $\sqrt{2}$ than a_n is? Why or why not? *[4]*

d. Give an adaptation of the method that will find approximations to $\sqrt{r}$ for any rational number r. *[3]*

8. Suppose A, B, and C are distinct points on a circle. Prove Thales' Theorem and its converse:

a. If AB is a diameter of the circle, then $\angle ACB$ is a right angle. *[5]*

b. If $\angle ACB$ is a right angle, then AB is a diameter of the circle. *[5]*

[Total = 70]

Part IV - Squares? Bonus!

$\square_{\square_{\square}}$**.** Write a poem touching on mathematics or its history. *[1]*

I hope that you enjoyed the course.
Have a great break!