TRENT UNIVERSITY
Mathematics-Science 380 – History of Mathematics
TAKE-HOME FINAL EXAMINATION
Due: 25 April, 2007

Instructions: Give complete answers to receive full credit, including references to any
and all sources you used. You may use your texts from this and any other courses, as well
as any handouts, class notes, and the like; you may also ask the instructor to clarify the
instructions or any of the questions; and you may use a calculator or computer to perform
any necessary calculations. *You may not consult any other sources, nor give or receive any
other aid on this exam, except with the intructor's explicit permission.*

**Part I – This and that.**   Do all three of **1 – 3**.

1. Answer all of **a** – **i**. *[9 = 9×1 each]*

   a. Who conjectured that every even integer greater than 2 is a sum of two prime
   numbers?

   b. Express 1/7 as a repeating sexagesimal.

   c. Can Fibonacci numbers be constructed by ruler and compass?

   d. Is there any evidence that the ancient Egyptians ever did mathematics for its
   own sake? If so, what is it?

   e. Who was the first mathematician to draw the graph of a function?

   f. What is the origin of the word *algebra*?

   g. Name three mathematicians who were also civil servants.

   h. Who discovered Pascal's triangle before Pascal?

   i. Who first denoted the ratio of a circle's circumference to its diameter by π?

2. Let $|PQ|$ denote the length of the line segment joining the points P and Q. Prove
Ptolemy's Theorem . . .

   *Suppose A, B, C, and D are any four points on a circle listed in clockwise order.
   Then $|AB| \cdot |CD| + |AD| \cdot |BC| = |AC| \cdot |BD|$.*

   . . . and show that the equation need not be true if the quadrilateral $ABCD$ is not
   inscribed in a circle. *[10]*
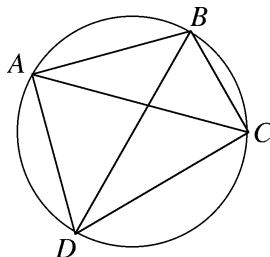
3. Which came first, pure or applied mathematics? Explain! *[7]*

Part II – History and Philosophy.   Do *one* of **4** or **5**. *[15]*

4. After calculus was introduced by Newton and Leibniz, mathematicians and scientists used and continued to develop it for nearly two centuries before it was given a more or less rigourous foundation. Explain why they did so, and argue whether or not they were justified in doing so.

5. To what extent was progress in mathematics before 1700 A.D. driven by progress in science and technology, and *vice versa*?

Part III – Mathematics.   Do any *two* of **6** – **9**. *[14 = 2×7 each]*

6. Suppose $f(x) = a_n x^n + a_{n-1} x^{n-1} + \cdots + a_1 x + a_0$ is a polynomial in x with integer coefficients. Can all the values $f(0)$, $f(1)$, $f(2)$, ... be prime numbers? If so, give an example of such a polynomial and show that it has this property; if not, prove that no polynomial in x with integer coefficients can have this property.

7. Prove the converse of the Pythagorean Theorem:

   If a triangle has sides of length a, b, and c, where $a^2 + b^2 = c^2$,
   then it has a right angle.

8. If u and v are two numbers and $u \geq v$, their *average* is $(u+v)/2$ and their *semidifference* is $(u - v)/2$. Do all three of **a** – **c**.

   a. Express uv in terms of the average and semidifference of u and v. *[1]*

   b. Given that $a = u + v$ and $b = uv$, solve for u and v in terms of a and b. *[3]*

   c. Use your solution to **b** to help solve the quadratic equation $x^2 + ax + b = 0$. *[3]*

9. Use the Dedekind cut or *schnitt* definition of real numbers to prove both **a** and **b**.

   a. The real numbers are *dense*, *i.e.* given any real numbers r and s with $r < s$, there is a real number t such that $r < t < s$. *[3]*

   b. Any set of real numbers with a lower bound has a greatest lower bound. *[4]*

[Total = 55]

Part IV - Fun.   Bonus!

$\circ^{\circ^{\circ}}$. Write a limerick touching on mathematics or its history. *[1]*

   Sylvester's Theorem
   (The Rank-Nullity Law)

A mathematician, Sylvester,
Had a wife he would often pester,
"As I raised the rank
All my null spaces shrank."
"Add them," she said, so he kissed her.

I HOPE YOU ENJOYED THE COURSE. ENJOY THE SUMMER!