

Mathematics-Science 380 – History of Mathematics
Trent University, 2006–2007

Assignment #11

Due in the week of 5 April, 2007.

Visiting the gaps

This assignment is concerned with a couple of results by mathematicians whose work we are mostly skipping over while looking at how calculus was eventually established on a rigorous footing.

Do *one* of **1** or **2**. As usual, you may work together and/or look things up, but please do remember to cite any help or sources you used.

- 1.** Prove that every integer $n \geq 0$ can be written as the sum of at most four (not necessarily different) integer squares. (This result was first proved by Adrien Marie Legendre.) [10]
- 2.** Prove that every integer $n \geq 1$ can be written as the sum of at most three (not necessarily different) triangular numbers. (This result was first proved by Carl Friedrich Gauss.) [10]

Note: This will be the last assignment. As agreed in class on 2007.03.19, instead of counting the best ten of twelve assignments for a total of 40% of the final mark, we will be counting the best nine of eleven, still for a total of 40% of the final mark.

Math is beautiful:

The mathematician's patterns, like the painter's or the poet's, must be *beautiful*; the ideas, like the colors or the words, must fit together in a harmonious way. . . . There is no permanent place in the world for ugly mathematics.

G. H. Hardy

This is how to tell:

The elegance of a theorem is directly proportional to the number of ideas you can see in it and inversely proportional to the effort it takes to see them.

George Polya

But it might be too much of a good thing:

Detest it [a certain difficult mathematics problem] just as much as lewd intercourse; it can deprive you of all your leisure, your health, your rest, and the whole happiness of your life.

Wolfgang Bolyai (From a letter to his son Janos.)