Trent University

Mathematics-Science 380 – History of Mathematics

Take-home Final Examination

Due: 29 April, 2005

Instructions: Give complete answers to receive full credit, including references to any and all sources you used. You may use (and refer to!) your text, any handouts you received, class notes, and any work that you submitted and had returned before completing this exam; you may also ask the instructor to clarify any of the questions; and you may use a calculator or computer to perform any necessary calculations. *You may not consult any other sources, nor give or receive any other aid on this exam, except with the intructor's explicit permission.*

Part I – This and that. Do **1** and *one* of **2** or **3**.

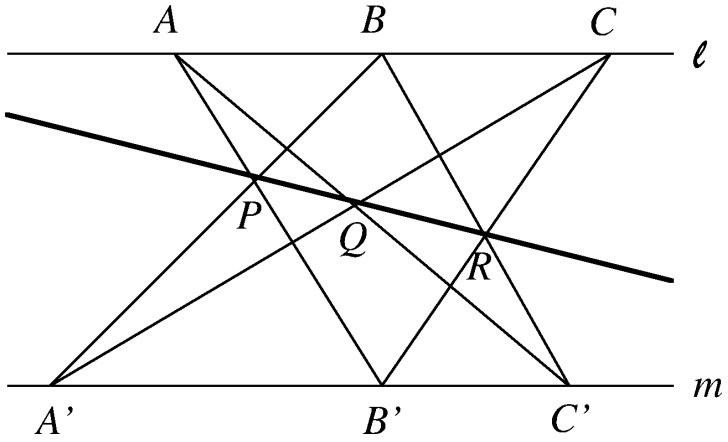
1. Answer all of **a** – **j**. *[10 = 10×1 each]*

- **a.** When was Archimedes' work *Method* rediscovered?
- **b.** Which famous mathematician is even better known nowadays as a poet?
- **c.** What are the three major interpretations of *Plimpton 322*?
- **d.** Who first systematically used a symbolic notation for algebraic expressions?
- **e.** Who got references from both Newton and Leibniz, but still couldn't get a job?
- **f.** Who developed the theory of proportion used in Euclidean geometry?
- **g.** Name three mathematicians who were also astrologers.
- **h.** How did the ancient Egyptians write fractions?
- **i.** Is Euclid's Fifth Postulate true or not?
- **j.** In which culture was mathematics often written in verse?

2. What is mathematics? Explain your answer! *[10]*

3. Prove the following special case of Pappus' Theorem. *[10]*

Suppose ℓ and m are distinct parallel lines, A, B, C are distinct points on ℓ, and A', B', C' are distinct points on m. Let $P = AB' \cap A'B$, $Q = AC' \cap A'C$, and $R = BC' \cap B'C$. Then P, Q, and R are collinear.



Part II – History. Do *one* of **4** or **5**.

4. "The feature distinguishing mathematics from the natural sciences is that its ultimate validating principle is proof rather than experience." Does the history of calculus tend to support this assertion or not? Justify your answer. *[15]*

5. The desire or need to solve practical problems and the pursuit of mathematics for its own sake have both played a big role in the history of mathematics. How did they interact in the development and evolution of geometry in ancient times? *[15]*

Part III – Mathematics. Do any *three* of **6** – **11**.

6. Prove that any integer $n > 1$ can be written as a product of one or more prime numbers. *[5]*

7. Prove that if n and m are the sums of two integer squares, then so is nm. *[5]*

8. Prove *Fermat's Little Theorem*: If p is a prime and $a > 0$ is an integer which is not a multiple of p, then $a^{p-1} - 1$ is a multiple of p. *[5]*

9. How many terms of *Gregory's series* $4 - \frac{4}{3} + \frac{4}{5} - \frac{4}{7} + \frac{4}{9} - \cdots$, which converges to π, must one add up to guarantee that the partial sum is within 0.01 of π? *[5]*

10. Prove that $u_n = \frac{1}{\sqrt{5}} \frac{\left(1+\sqrt{5}\right)^n - \left(1-\sqrt{5}\right)^n}{2^n}$, the nth Fibonacci number, is the closest integer to $\frac{1}{\sqrt{5}} \left(\frac{1+\sqrt{5}}{2}\right)^n$. *[5]*

11. Given a line segment of length 1 as a reference, give a ruler and compass construction of a line segment of the length of the *golden section*, $\varphi = \frac{-1+\sqrt{5}}{2}$. *[5]*

[Total = 50]

Part IV - Fun. Bonus!

$6\times9 = 42_{13}$. Write a *haiku* touching on mathematics or its history. *[1]*

haiku?

seventeen in three:
five and seven and five of
syllables in lines

HAVE FUN THIS SUMMER!