

## ΣΤΟΙΧΕΙΩΝ α'

### “Οροι

- α' Σημεῖόν ἐστιν, οὗ μέρος οὐθέν.
- β' Γραμμὴ δὲ μῆκος ἀπλατές.
- γ' Γραμμῆς δὲ πέρατα σημεῖα.
- δ' Εὐθεῖα γραμμὴ ἐστίν, ἣτις ἐξ ἴσου τοῖς ἐφ' ἑαυτῆς σημείοις κεῖται.
- ε' Ἐπιφάνεια δὲ ἐστίν, ἧ μῆκος καὶ πλάτος μόνον ἔχει.
- ς' Ἐπιφανείας δὲ πέρατα γραμμαί.
- ζ' Ἐπίπεδος ἐπιφάνειά ἐστίν, ἣτις ἐξ ἴσου ταῖς ἐφ' ἑαυτῆς εὐθείαις κεῖται.
- η' Ἐπίπεδος δὲ γωνία ἐστίν ἢ ἐν ἐπιπέδῳ δύο γραμμῶν ἀπτομένων ἀλλήλων καὶ μὴ ἐπ' εὐθείας κειμένων πρὸς ἀλλήλας τῶν γραμμῶν κλίσις.
- θ' Ὄταν δὲ αἱ περιέχουσαι τὴν γωνίαν γραμμαί εὐθεῖαι ᾧσιν, εὐθύγραμμος καλεῖται ἡ γωνία.
- ι' Ὄταν δὲ εὐθεῖα ἐπ' εὐθεῖαν σταθεῖσα τὰς ἐφεξῆς γωνίας ἴσας ἀλλήλαις ποιῇ, ὀρθὴ ἑκατέρα τῶν ἴσων γωνιῶν ἐστί, καὶ ἡ ἐφεστηκυῖα εὐθεῖα κάθετος καλεῖται, ἐφ' ἣν ἐφέστηκεν.
- ια' Ἀμβλεῖα γωνία ἐστίν ἢ μείζων ὀρθῆς.
- ιβ' Ὄξεῖα δὲ ἢ ἐλάσσων ὀρθῆς.
- ιγ' Ὄρος ἐστίν, ὃ τινός ἐστι πέρας.
- ιδ' Σχῆμά ἐστι τὸ ὑπὸ τινος ἢ τινων ὄρων περιεχόμενον.
- ιε' Κύκλος ἐστὶ σχῆμα ἐπίπεδον ὑπὸ μιᾶς γραμμῆς περιεχόμενον [ἢ καλεῖται περιφέρεια], πρὸς ἣν ἀφ' ἑνὸς σημείου τῶν ἐντὸς τοῦ σχήματος κειμένων πᾶσαι αἱ προσπίπτουσαι εὐθεῖαι [πρὸς τὴν τοῦ κύκλου περιφέρειαν] ἴσαι ἀλλήλαις εἰσίν.
- ισ' Κέντρον δὲ τοῦ κύκλου τὸ σημεῖον καλεῖται.
- ις' Διάμετρος δὲ τοῦ κύκλου ἐστίν εὐθεῖα τις διὰ τοῦ κέντρου ἠγμένη καὶ περατουμένη ἐφ' ἑκάτερα τὰ μέρη ὑπὸ τῆς τοῦ κύκλου περιφερείας, ἣτις καὶ δίχα τέμνει τὸν κύκλον.
- ιη' Ἡμικύκλιον δὲ ἐστὶ τὸ περιεχόμενον σχῆμα ὑπὸ τε τῆς διαμέτρου καὶ τῆς ἀπολαμβανομένης ὑπ' αὐτῆς περιφερείας. κέντρον δὲ τοῦ ἡμικυκλίου τὸ αὐτό, ὃ καὶ τοῦ κύκλου ἐστίν.
- ιθ' Σχήματα εὐθύγραμμά ἐστί τὰ ὑπὸ εὐθειῶν περιεχόμενα, τρίπλευρα μὲν τὰ ὑπὸ τριῶν, τετράπλευρα δὲ τὰ ὑπὸ τεσσάρων, πολύπλευρα δὲ τὰ ὑπὸ πλειόνων ἢ τεσσάρων εὐθειῶν περιεχόμενα.

# ELEMENTS BOOK 1

## Definitions

- 1 A point is that of which there is no part.
- 2 And a line is a length without breadth.
- 3 And the extremities of a line are points.
- 4 A straight-line is whatever lies evenly with points upon itself.
- 5 And a surface is that which has length and breadth alone.
- 6 And the extremities of a surface are lines.
- 7 A plane surface is whatever lies evenly with straight-lines upon itself.
- 8 And a plane angle is the inclination of the lines, when two lines in a plane meet one another, and are not laid down straight-on with respect to one another.
- 9 And when the lines containing the angle are straight then the angle is called rectilinear.
- 10 And when a straight-line stood upon (another) straight-line makes adjacent angles (which are) equal to one another, each of the equal angles is a right-angle, and the former straight-line is called perpendicular to that upon which it stands.
- 11 An obtuse angle is greater than a right-angle.
- 12 And an acute angle is less than a right-angle.
- 13 A boundary is that which is the extremity of something.
- 14 A figure is that which is contained by some boundary or boundaries.
- 15 A circle is a plane figure contained by a single line [which is called a circumference], (such that) all of the straight-lines radiating towards [the circumference] from a single point lying inside the figure are equal to one another.
- 16 And the point is called the center of the circle.
- 17 And a diameter of the circle is any straight-line, being drawn through the center, which is brought to an end in each direction by the circumference of the circle. And any such (straight-line) cuts the circle in half.<sup>1</sup>
- 18 And a semi-circle is the figure contained by the diameter and the circumference it cuts off. And the center of the semi-circle is the same (point) as (the center of) the circle.
- 19 Rectilinear figures are those figures contained by straight-lines: trilateral figures being contained by three straight-lines, quadrilateral by four, and multilateral by more than four.

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<sup>1</sup>This should really be counted as a postulate, rather than as part of a definition.

## ΣΤΟΙΧΕΙΩΝ α'

- κ' Τῶν δὲ τριπλεύρων σχημάτων ἰσόπλευρον μὲν τρίγωνόν ἐστι τὸ τὰς τρεῖς ἴσας ἔχον πλευράς, ἰσοσκελὲς δὲ τὸ τὰς δύο μόνας ἴσας ἔχον πλευράς, σιαληνὸν δὲ τὸ τὰς τρεῖς ἀνίσους ἔχον πλευράς.
- κα' Ἐπι δὲ τῶν τριπλεύρων σχημάτων ὀρθογώνιον μὲν τρίγωνόν ἐστι τὸ ἔχον ὀρθὴν γωνίαν, ἄμβλυγώνιον δὲ τὸ ἔχον ἄμβλειαν γωνίαν, ὀξυγώνιον δὲ τὸ τὰς τρεῖς ὀξείας ἔχον γωνίας.
- κβ' Τῶν δὲ τετραπλεύρων σχημάτων τετράγωνον μὲν ἐστίν, ὃ ἰσόπλευρόν τε ἐστὶ καὶ ὀρθογώνιον, ἑτερόμηκες δέ, ὃ ὀρθογώνιον μὲν, οὐκ ἰσόπλευρον δέ, ῥόμβος δέ, ὃ ἰσόπλευρον μὲν, οὐκ ὀρθογώνιον δέ, ῥομβοειδὲς δὲ τὸ τὰς ἀπεναντίον πλευράς τε καὶ γωνίας ἴσας ἀλλήλαις ἔχον, ὃ οὔτε ἰσόπλευρόν ἐστίν οὔτε ὀρθογώνιον· τὰ δὲ παρὰ ταῦτα τετράπλευρα τραπέζια καλεῖσθω.
- κγ' Παράλληλοί εἰσιν εὐθεῖαι, αἵτινες ἐν τῷ αὐτῷ ἐπιπέδῳ οὔσαι καὶ ἐκβαλλόμεναι εἰς ἄπειρον ἐφ' ἑκάτερα τὰ μέρη ἐπὶ μηδέτερα συμπίπτουσιν ἀλλήλαις.

## Αἰτήματα

- α' Ἡιτήσθω ἀπὸ παντὸς σημείου ἐπὶ πᾶν σημεῖον εὐθεῖαν γραμμὴν ἀγαγεῖν.
- β' Καὶ πεπερασμένην εὐθεῖαν κατὰ τὸ συνεχὲς ἐπ' εὐθείας ἐκβαλεῖν.
- γ' Καὶ παντὶ κέντρῳ καὶ διαστήματι κύκλον γράφεισθαι.
- δ' Καὶ πάσας τὰς ὀρθὰς γωνίας ἴσας ἀλλήλαις εἶναι.
- ε' Καὶ ἐὰν εἰς δύο εὐθείας εὐθεῖα ἐμπίπτουσα τὰς ἐντὸς καὶ ἐπὶ τὰ αὐτὰ μέρη γωνίας δύο ὀρθῶν ἐλάσσονας ποιῇ, ἐκβαλλομένης τὰς δύο εὐθείας ἐπ' ἄπειρον συμπίπτειν, ἐφ' ἃ μέρη εἰσὶν αἱ τῶν δύο ὀρθῶν ἐλάσσονες.

## Κοινὰ ἔννοιαι

- α' Τὰ τῷ αὐτῷ ἴσα καὶ ἀλλήλοις ἐστὶν ἴσα.
- β' Καὶ ἐὰν ἴσοις ἴσα προστεθῇ, τὰ ὅλα ἐστὶν ἴσα.
- γ' Καὶ ἐὰν ἀπὸ ἴσων ἴσα ἀφαιρεθῇ, τὰ καταλειπόμενά ἐστὶν ἴσα.
- δ' Καὶ τὰ ἐφαρμόζοντα ἐπ' ἀλλήλα ἴσα ἀλλήλοις ἐστὶν.
- ε' Καὶ τὸ ὅλον τοῦ μέρους μεῖζόν [ἐστίν].

## ELEMENTS BOOK 1

- 20 And of the trilateral figures: an equilateral triangle is that having three equal sides, an isosceles (triangle) that having only two equal sides, and a scalene (triangle) that having three unequal sides.
- 21 And further of the trilateral figures: a right-angled triangle is that having a right-angle, an obtuse-angled (triangle) that having an obtuse angle, and an acute-angled (triangle) that having three acute angles.
- 22 And of the quadrilateral figures: a square is that which is right-angled and equilateral, a rectangle that which is right-angled but not equilateral, a rhombus that which is equilateral but not right-angled, and a rhomboid that having opposite sides and angles equal to one another which is neither right-angled nor equilateral. And let quadrilateral figures besides these be called trapezia.
- 23 Parallel lines are straight-lines which, being in the same plane, and being produced to infinity in each direction, meet with one another in neither (of these directions).

### Postulates

- 1 Let it have been postulated to draw a straight-line from any point to any point.
- 2 And to produce a finite straight-line continuously in a straight-line.
- 3 And to draw a circle with any center and radius.
- 4 And that all right-angles are equal to one another.
- 5 And that if a straight-line falling across two (other) straight-lines makes internal angles on the same side (of itself) less than two right-angles, being produced to infinity, the two (other) straight-lines meet on that side (of the original straight-line) that the (internal angles) are less than two right-angles (and do not meet on the other side).<sup>2</sup>

### Common Notions

- 1 Things equal to the same thing are also equal to one another.
- 2 And if equal things are added to equal things then the wholes are equal.
- 3 And if equal things are subtracted from equal things then the remainders are equal.<sup>3</sup>
- 4 And things coinciding with one another are equal to one another.
- 5 And the whole [is] greater than the part.

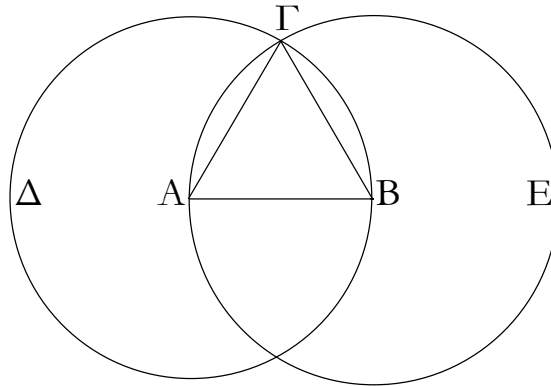
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<sup>2</sup>This postulate effectively specifies that we are dealing with the geometry of *flat*, rather than curved, space.

<sup>3</sup>As an obvious extension of C.N.s 2 & 3—if equal things are added or subtracted from the two sides of an inequality then the inequality remains an inequality of the same type.

# ΣΤΟΙΧΕΙΩΝ α'

α'



Ἐπὶ τῆς δοθείσης εὐθείας πεπερασμένης τρίγωνον ἰσόπλευρον συστήσασθαι.

Ἐστω ἡ δοθεῖσα εὐθεῖα πεπερασμένη ἡ  $AB$ .

Δεῖ δὴ ἐπὶ τῆς  $AB$  εὐθείας τρίγωνον ἰσόπλευρον συστήσασθαι.

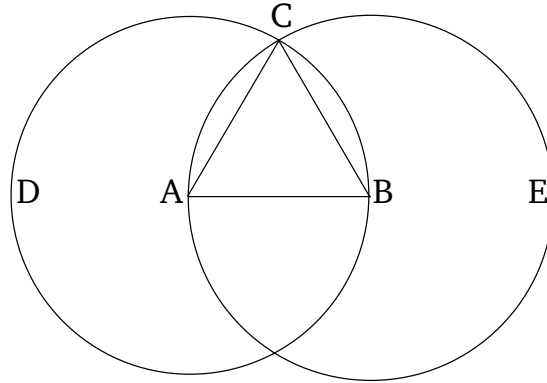
Κέντρῳ μὲν τῷ  $A$  διαστήματι δὲ τῷ  $AB$  κύκλος γεγράφθω ὁ  $BΓΔ$ , καὶ πάλιν κέντρῳ μὲν τῷ  $B$  διαστήματι δὲ τῷ  $BA$  κύκλος γεγράφθω ὁ  $ΑΓΕ$ , καὶ ἀπὸ τοῦ  $Γ$  σημείου, καθ' ὃ τέμνουσιν ἀλλήλους οἱ κύκλοι, ἐπὶ τὰ  $A, B$  σημεῖα ἐπεζεύχθωσαν εὐθεῖαι αἱ  $ΓΑ, ΓΒ$ .

Καὶ ἐπεὶ τὸ  $A$  σημεῖον κέντρον ἐστὶ τοῦ  $ΓΔΒ$  κύκλου, ἴση ἐστὶν ἡ  $ΑΓ$  τῇ  $AB$ . πάλιν, ἐπεὶ τὸ  $B$  σημεῖον κέντρον ἐστὶ τοῦ  $ΓΑΕ$  κύκλου, ἴση ἐστὶν ἡ  $ΒΓ$  τῇ  $BA$ . ἐδείχθη δὲ καὶ ἡ  $ΓΑ$  τῇ  $AB$  ἴση· ἑκατέρα ἄρα τῶν  $ΓΑ, ΓΒ$  τῇ  $AB$  ἐστὶν ἴση. τὰ δὲ τῷ αὐτῷ ἴσα καὶ ἀλλήλοις ἐστὶν ἴσα· καὶ ἡ  $ΓΑ$  ἄρα τῇ  $ΓΒ$  ἐστὶν ἴση· αἱ τρεῖς ἄρα αἱ  $ΓΑ, AB, ΒΓ$  ἴσαι ἀλλήλαις εἰσίν.

ἰσόπλευρον ἄρα ἐστὶ τὸ  $ΑΒΓ$  τρίγωνον. καὶ συνέσταται ἐπὶ τῆς δοθείσης εὐθείας πεπερασμένης τῆς  $AB$ · ὅπερ ἔδει ποιῆσαι.

# ELEMENTS BOOK 1

## Proposition 1



To construct an equilateral triangle on a given finite straight-line.

Let  $AB$  be the given finite straight-line.

So it is required to construct an equilateral triangle on the straight-line  $AB$ .

Let the circle  $BCD$  with center  $A$  and radius  $AB$  have been drawn [Post. 3], and again let the circle  $ACE$  with center  $B$  and radius  $BA$  have been drawn [Post. 3]. And let the straight-lines  $CA$  and  $CB$  have been joined from the point  $C$ , where the circles cut one another,<sup>4</sup> to the points  $A$  and  $B$  (respectively) [Post. 1].

And since the point  $A$  is the center of the circle  $CDB$ ,  $AC$  is equal to  $AB$  [Def. 1.15]. Again, since the point  $B$  is the center of the circle  $CAE$ ,  $BC$  is equal to  $BA$  [Def. 1.15]. But  $CA$  was also shown (to be) equal to  $AB$ . Thus,  $CA$  and  $CB$  are each equal to  $AB$ . But things equal to the same thing are also equal to one another [C.N. 1]. Thus,  $CA$  is also equal to  $CB$ . Thus, the three (straight-lines)  $CA$ ,  $AB$ , and  $BC$  are equal to one another.

Thus, the triangle  $ABC$  is equilateral, and has been constructed on the given finite straight-line  $AB$ . (Which is) the very thing it was required to do.

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<sup>4</sup>The assumption that the circles do indeed cut one another should be counted as an additional postulate. There is also an implicit assumption that two straight-lines cannot share a common segment.