Mathematics 3790H – Analysis I: Introduction to analysis
TRENT UNIVERSITY, Winter 2015

Quizzes

Quiz #1. *Tuesday, 13 January, 2015. [5 minutes]*

1. Use the ε-δ definition of limits to verify that $\lim_{x \to 3} (41 - 13x) = 2$. *[5]*

Quiz #2. *Tuesday, 20 January, 2015. [10 minutes]*

1. Suppose $A \cap B \subseteq \mathbb{R}$ and $A \cap B \neq \emptyset$. Show that $\sup(A \cap B)$ is less than or equal to to both $\sup(A)$ and $\sup(B)$. *[5]*

Quiz #3. *Tuesday, 27 January, 2015. [10 minutes]*

1. Consider the sequence $a_n = 1 + \dfrac{(-1)^n}{n}$, where $n \geq 0$. Find a monotonic subsequence of $\{a_n\}$ and find the limit of this subsequence. *[5]*

Quiz #4. ~~*Tuesday, 3*~~ *Wednesday, 4 February, 2015. [10 minutes]*

1. Let $a_n = \sin\left(\frac{n\pi}{4}\right)$ for $n \geq 0$. Find $\liminf_{n \to \infty} a_n$. *[5]*

Quiz #5. *Tuesday, 10 February, 2015. [10 minutes]*

1. Suppose $\sum_{k=0}^{\infty} a_k$ is a convergent series and $b_n = \sum_{k=n}^{\infty} a_k$ for each $n \geq 0$. Show that $\lim_{n \to \infty} b_n = 0$. *[5]*

Quiz #5. *Tuesday, 10 February, 2015. [10 minutes]*

1. Suppose $\sum_{n=0}^{\infty} a_n$ converges and $a_n \geq 0$ for all $n \geq 0$, and $\{b_n\}$ is a bounded sequence, *i.e.* for some $B \in \mathbb{R}$, $|b_n| \leq B$ for all $n \geq 0$. Show that $\sum_{n=0}^{\infty} a_n b_n$ also converges. *[5]*

Quiz #6. *Tuesday, 24 February, 2015. [10 minutes]*

1. Suppose $\{a_k\}$ is a sequence, and let $b_n = \sum_{k=n}^{\infty} a_k$ for each $n \geq 0$. Show that if $\lim_{n \to \infty} b_n = 0$, then $\sum_{n=0}^{\infty} a_n$ converges. *[5]*

Quiz #7. ~~*Tuesday, 3*~~ *Wednesday, 4 March, 2015. [10 minutes]*

1. Determine whether $\sum_{n=2}^{\infty} \dfrac{1}{n \ln(n)}$ converges or diverges. *[5]*

Quiz #8. *Tuesday, 10 March, 2015. [10 minutes]*

1. Show that $f(x) = x + 1$ is uniformly continuous on $[0, 1]$. *[5]*

Quiz #9. *Tuesday, 17 March, 2015. [10 minutes]*

Recall that $f_n \underset{\text{unif}}{\longrightarrow} f$ on an interval I if for every $\varepsilon > 0$ there is an N such that for all $n \geq N$ and all $x \in I$, $|f_n(x) - f(x)| < \varepsilon$.

1. Suppose $f_n(x) = \dfrac{\sin(nx)}{n}$ for $n \geq 1$ and $f(x) = 0$ for $-\infty < x < \infty$. Verify that $f_n \underset{\text{unif}}{\longrightarrow} f$ on $(-\infty, \infty)$. *[5]*

Quiz #10. *Tuesday, 24 March, 2015. [10 minutes]*

1. Show that $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n^2} \arctan\left(\frac{x}{n}\right)$ converges uniformly on $(0, 1)$. *[5]*

Quiz #10. *Alternate Version. [10 minutes]*

1. Show that $\displaystyle\sum_{n=1}^{\infty} \frac{\left(1 - \frac{x}{n}\right)^n}{2^n}$ converges uniformly on $[0, 1]$. *[5]*