Mathematics 3790H – Analysis I: Real analysis
Trent University, Winter 2015

Assignment #9
Uniform convergence
Due on Friday, 20 March, 2015.

Recall from class and the textbook that a sequence of functions $\{f_n\}$ *converges uniformly* to a function f on an interval I if for any $\varepsilon > 0$ there is an N such that for all $n \geq N$ and all $x \in I$, $|f_n(x) - f(x)| < \varepsilon$. (This is often denoted by something like $f_n \underset{\text{unif}}{\longrightarrow} f$ or $f_n \to f$ [unif]).

1. Suppose f is a function which is defined and uniformly continuous on $(-\infty, \infty)$. Let $f_n(x) = f\left(x + \frac{1}{n}\right)$. Show that $f_n \underset{\text{unif}}{\longrightarrow} f$ on $(-\infty, \infty)$. *[7]*

2. Is the result stated in **1** true if one replaces "uniformly continuous" by " continuous"? Prove that it is or find a counterexample. *[3]*