Mathematics 3790H – Analysis I: Real analysis
Trent University, Winter 2015

Assignment #5
Due on Friday, 13 February, 2015.

1. Let $\{a_n\}$ be a sequence with $a_n \geq 0$ for all $n \geq 0$, and $u_n = \sup\{\, a_n,\, a_{n+1},\, a_{n+2},\, \cdots \}$ for each $n \geq 0$. Show that if $\displaystyle\sum_{n=0}^{\infty} u_n$ converges, then so does $\displaystyle\sum_{n=0}^{\infty} a_n$. *[5]*

2. Let $\{a_n\}$ be a sequence with $a_n \geq 0$ for all $n \geq 0$, and $u_n = \sup\{\, a_n,\, a_{n+1},\, a_{n+2},\, \cdots \}$ and $\ell_n = \inf\{\, a_n,\, a_{n+1},\, a_{n+2},\, \cdots \}$ for each $n \geq 0$. Is it true that if $\displaystyle\sum_{n=0}^{\infty} u_n$ and $\displaystyle\sum_{n=0}^{\infty} \ell_n$ both converge, then so does $\displaystyle\sum_{n=0}^{\infty} a_n$? Prove it or give a counterexample. *[5]*