

Mathematics 3790H – Analysis I: Real analysis
TRENT UNIVERSITY, Winter 2015

Assignment #11
An evil Taylor series
Due on Friday, 3 April, 2015.

Let $f(x) = \begin{cases} e^{-1/x^2} & x \neq 0 \\ 0 & x = 0 \end{cases}$, then $f(x)$ is defined and continuous for all $x \in \mathbb{R}$.

1. Show that

$$\frac{d^n}{dx^n} f(x) = f^{(n)}(x) = p_{3n}\left(\frac{1}{x}\right) e^{-1/x^2}$$

for all $x \neq 0$ and all $n \geq 1$, where $p_{3n}(t)$ is a polynomial of degree $3n$ in t . [4]

HINT: This is probably best done by induction on n .

2. Show that $f^{(n)}(0) = 0$ for all $n \geq 1$. [4]

3. What is the Taylor series of $f(x)$ at 0? What is its radius of convergence? [1]

4. For what x is $f(x)$ equal to its Taylor series at 0? [1]

NOTE: This function is the poster child for the fact that a Taylor series for a function does not always have to converge to the function it came from ...