

**Mathematics 3790H – Analysis I: Real analysis**  
TRENT UNIVERSITY, Winter 2015

**Assignment #10**  
**More uniform convergence**  
*Due on Friday, 27 March, 2015.*

Recall from class and the textbook that a series of functions  $\sum_{k=0}^{\infty} f_k(x)$  converges uniformly to a function  $S(x)$  on an interval  $I$  if for any  $\varepsilon > 0$  there is an  $N$  such that for all  $n \geq N$  and all  $x \in I$ ,  $|S_n(x) - S(x)| < \varepsilon$ , where  $S_n(x) = \sum_{k=0}^n f_k(x)$  for each  $n \geq 0$ .

1. Suppose that for all  $k \geq 0$  and  $x \in I$ ,  $|f_k(x)| \leq a_k$ , and that  $\sum_{n=0}^{\infty} a_k$  converges. Show that  $\sum_{k=0}^{\infty} f_k(x)$  converges uniformly on  $I$ . [10]