Mathematics 3790H – Analysis I: Introduction to analysis
Trent University, Winter 2014

Quizzes

Quiz #1. Tuesday, 14 January, 2014. *[10 minutes]*

1. Suppose you are given that $\inf\left\{ \frac{1}{n} \mid n \in \mathbb{N} \text{ and } n > 0 \right\} = 0$. Use this fact to help prove the Archimedean Property of $\mathbb{R}$, *i.e.* that $\mathbb{N}$ has no upper bound in $\mathbb{R}$. *[5]*

Quiz #2. Tuesday, 21 January, 2014. *[10 minutes]*

1. Suppose that $\{s_n\}$ and $\{t_n\}$ are sequences with $\lim_{n\to\infty} s_n = 3$ and $\lim_{n\to\infty} (t_n - s_n) = 0$. Use the ε–N definition of the limit of a sequence to show that $\lim_{n\to\infty} t_n = 3$, too. *[5]*

Quiz #3. Tuesday, 28 January, 2014. *[10 minutes]*

1. Show that $\lim_{n\to\infty} \dfrac{\cos(n)}{n} = 0$. [With or without epsilonics – your choice!] *[5]*

Quiz #4. Tuesday, 4 February, 2014. *[10 minutes]*

1. Suppose that $\{s_{n_k}\}$ is a subsequence of the sequence $\{s_n\}$. Which of

$$\limsup_{n\to\infty} s_n \geq \limsup_{k\to\infty} s_{n_k} \quad \text{or} \quad \limsup_{n\to\infty} s_n \leq \limsup_{k\to\infty} s_{n_k}$$

must be true? Explain why! *[5]*

Quiz #5. Tuesday, 11 February, 2014. *[10 minutes]*

1. Suppose $\sum_{n=1}^{\infty} |a_n|$ is a convergent series. Show that $\sum_{n=1}^{\infty} a_n$ converges as well. *[5]*

Quiz #6. Tuesday, 25 February, 2014. *[10 minutes]*

1. Suppose $\displaystyle\sum_{k=1}^{\infty} a_k$ converges. Does it follow that $\displaystyle\sum_{k=1}^{n} k a_k$ is bounded for all n? Prove it or give a counterexample. *[5]*

Quiz #7. Tuesday, 4 March, 2014. *[10 minutes]*

1. It turns out that $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n} = 1 - \frac{1}{2} + \frac{1}{3} - \frac{1}{4} + \frac{1}{5} - \frac{1}{6} + \cdots = \ln(2)$. If so, what is $\displaystyle\sum_{k=0}^{\infty} \left[\frac{1}{2k+1} - \frac{1}{4k+2} - \frac{1}{4k+4} \right] = 1 - \frac{1}{2} - \frac{1}{4} + \frac{1}{3} - \frac{1}{6} - \frac{1}{8} + \frac{1}{5} - \frac{1}{10} - \frac{1}{12} + \ldots$? *[5]*

Quiz #8. Tuesday, 11 March, 2014. *[10 minutes]*

1. For each $n \geq 0$, let $f_n : \mathbb{R} \to \mathbb{R}$ be defined by $f_n(x) = \arctan(nx)$. Determine the function $f : \mathbb{R} \to \mathbb{R}$ that is the pointwise limit of the f_n (*i.e.* such that $f_n(x) \to f(x)$ for each $x \in \mathbb{R}$), and whether it is continuous or not. *[5]*

Quiz #9. Tuesday, 18 March, 2014. *[10 minutes]*

1. Suppose that for $n \geq 0$, $p_n(x) = a_n x^n$ for a sequence $\{a_n\}$ of positive real numbers such that $\lim_{n \to 0} a_n = 0$, and let $\zeta(x) = 0$ for all x. Show that $p_n \underset{\text{unif}}{\longrightarrow} \zeta$ on $[-1, 1]$. *[5]*

Quiz #10. Tuesday, 25 March, 2014. *[15 minutes]*

1. Find a power series equal to $f(x) = \dfrac{1}{(1-x)^2}$ (when it converges) and determine its interval of convergence. *[5]*

   Hint: $\dfrac{d}{dx}\left(\dfrac{1}{1-x}\right) = \dfrac{1}{(1-x)^2}$. Mind you, there is at least one completely different way to get the series ...

Quiz #11. Wednesday, 2 April, 2014. *[20 minutes]*

1. Use Taylor's formula to find the Taylor series at 0 of $f(x) = e^{x/2}$. *[3]*
2. Show that the Taylor series at 0 of $f(x)$ converges (pointwise) to $f(x)$ for all x. *[2]*

   Hint: The Lagrange form of the nth remainder term for the Taylor series at 0 is $R_n(x) = \frac{f^{(n)}(t)}{n!} x^n$, where t is between 0 and x.