Mathematics 3790H – Analysis I: Introduction to analysis
TRENT UNIVERSITY, Winter 2014

Assignment #9
Uniqueness and Taylor series
Due on Friday, 21 March, 2014.

Recall that the Taylor series at a of a function $f(x)$ is the power series:

$$\sum_{n=0}^{\infty} \frac{f^{(n)}(a)}{n!}(x-a)^n$$

Of course, this requires the function $f(x)$ and all of its derivatives to be defined at a. It is clear from the formula that a function can have only one Taylor series, assuming it has one at all. It is a question of some interest whether a given power series can serve as the Taylor series for two different functions. (Given how often Taylor polynomials are used to approximate functions for computational purposes, for example, it would be nice to know that we are not accidentally going to compute approximations to some different function.) Your task in this assignment will be to answer this question.

1. Suppose $\sum_{n=0}^{\infty} a_n(x-a)^n$ is the Taylor series at a for two functions $f(x)$ and $g(x)$. Prove that $f(x) = g(x)$, at least for x near a, or find a counterexample showing that $f(x)$ need not be equal to $g(x)$ except at $x = a$. *[10]*