

**Mathematics 3790H – Analysis I: Introduction to analysis**  
TRENT UNIVERSITY, Winter 2014

**Assignment #7**

*Due on Friday, 7 March, 2014.*

1. Prove that the partial sums of  $\sum_{n=0}^{\infty} \sin(n)$  are bounded. [5]
2. Use the result stated in 1 to help show that  $\sum_{n=2}^{\infty} \frac{\sin(n)}{\ln(n)}$  converges. [1]
3. Suppose  $m$  is any real number and  $x$  is a real variable. Then the *binomial series* for  $m$  is

$$\begin{aligned} & 1 + mx + \frac{m(m-1)}{2!}x^2 + \frac{m(m-1)(m-2)}{3!}x^3 + \dots \\ &= 1 + \sum_{k=1}^{\infty} \frac{m(m-1)\cdots(m-k+1)}{k!}x^k. \end{aligned}$$

Show that if  $|x| < 1$ , then the binomial series converges for all  $m \in \mathbb{R}$ . [4]

NOTE: The binomial series for  $m$  is the Taylor series at 0 of  $f(x) = (1+x)^m$ .