Mathematics 3790H – Analysis I: Introduction to analysis
Trent University, Winter 2012

Quizzes

Quiz #1. ~~Monday, 16~~ Thursday, 19 January, 2012. *[10 minutes]*

1. Suppose $X \subset \mathbb{R}$ has $\sup(X) = \text{lub}(X) = a$. Show that if $Y = \{\, -x \mid x \in X \,\}$, then $\inf(Y) = \text{glb}(Y) = -a$. *[5]*

Quiz #2. ~~Monday, 23~~ Wednesday, 25 January, 2012. *[10 minutes]*

1. Suppose you are given that $\displaystyle\lim_{n\to\infty} \frac{1}{n} = 0$. Use this fact, plus some algebra and the limit laws for sequences, to compute $\displaystyle\lim_{n\to\infty} \frac{n^2+2n+1}{n^2+2n+2}$. *[5]*

Quiz #3. Monday, 30 January, 2012. *[10 minutes]*

1. If n is a positive integer, then the *square-free part of* n is $\upsilon(n) = \dfrac{n}{m^2}$, where m is the largest positive integer whose square divides n. Let $a_n = \dfrac{1}{\upsilon(n)}$ for $n \geq 1$. Find two subsequences of a_n which converge to different limits. *[5]*

Hint: The first thirty elements of the sequence are:

n	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
$\upsilon(n)$	1	2	3	1	5	6	7	2	1	10	11	3	13	14	15
a_n	1	$\frac{1}{2}$	$\frac{1}{3}$	1	$\frac{1}{5}$	$\frac{1}{6}$	$\frac{1}{7}$	$\frac{1}{2}$	1	$\frac{1}{10}$	$\frac{1}{11}$	$\frac{1}{3}$	$\frac{1}{13}$	$\frac{1}{14}$	$\frac{1}{15}$

n	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30
$\upsilon(n)$	1	17	2	19	5	21	22	23	6	1	26	3	28	29	30
a_n	1	$\frac{1}{17}$	$\frac{1}{2}$	$\frac{1}{19}$	$\frac{1}{5}$	$\frac{1}{21}$	$\frac{1}{22}$	$\frac{1}{23}$	$\frac{1}{6}$	1	$\frac{1}{26}$	$\frac{1}{3}$	$\frac{1}{7}$	$\frac{1}{29}$	$\frac{1}{30}$

Quiz #4. Monday, 6 February, 2012. *[10 minutes]*

1. Determine whether $\displaystyle\sum_{n=0}^{\infty} \frac{n}{n^2-n+1}$ converges or not. *[5]*

Quiz #5. Monday, 13 February, 2012. *[10 minutes]*

1. Determine whether $\displaystyle\sum_{n=0}^{\infty} \frac{n^2+n}{3^n}$ converges or not. *[5]*

Quiz #6. Monday, 27 February, 2012. *[10 minutes]*

1. Find the radius and interval of convergence of the power series

$$\sum_{n=1}^{\infty} \frac{\frac{1}{2}\left(\frac{1}{2}-1\right)\left(\frac{1}{2}-2\right)\cdots\left(\frac{1}{2}-n+1\right)}{n!} x^n . \quad [5]$$

Quiz #7. Monday, 5 March, 2012. *[10 minutes]*

1. Verify that the sequence of functions $f_n(x) = e^{-nx}$ converges uniformly to the function $f(x) = 0$ on the interval $[1, 2]$. *[5]*

Quiz #8. Monday, 12 March, 2012. *[15 minutes]*

1. Give an example of a sequence of continuous functions $f_n(x)$ defined on a closed interval $[a, b]$ such that $c_n = \int_a^b f_n(x)\,dx$ converges to some real number c, but $f_n(x)$ does not converge uniformly on $[a, b]$. *[5]*

Quiz #9. Monday, 19 March, 2012. *[10 minutes]*

1. Find the radius of convergence of $\displaystyle\sum_{n=0}^{\infty} \frac{(-1)^n}{(2n+1)!} x^{2n+1}$. *[2]*

2. Assuming that $\sin(x) = \displaystyle\sum_{n=0}^{\infty} \frac{(-1)^n}{(2n+1)!} x^{2n+1}$ when the series converges, find a power series equal to $\cos(x)$. *[3]*

Quiz #10. Monday, 26 March, 2012. *[15 minutes]*

1. Find the Taylor series at 0 of $f(x) = \dfrac{1}{1-7x}$ and determine its interval of convergence. *[5]*

Quiz #11. Monday, 2 April, 2012. *[15 minutes]*

Recall that the Taylor series at 0 of $f(x) = \ln(1+x)$ is $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n} x^n$ and has radius of convergence $r = 1$.

1. What does k need to be to ensure that $\displaystyle\sum_{n=1}^{k} \frac{(-1)^{n+1}}{n} \left(-\frac{1}{2}\right)^n$ is within $\dfrac{1}{32}$ of $\ln\left(\dfrac{1}{2}\right) = \ln\left(1 - \dfrac{1}{2}\right)$? *[5]*

Hint: Use the Lagrange form of the remainder.