Mathematics 3790H – Analysis I: Introduction to analysis
TRENT UNIVERSITY, Fall 2009

Assignment #5
Due: Thursday, 26 November, 2009

Cauchy sequences

The counterpart for sequences of the Cauchy Convergence Criterion for series is the following notion:

> DEFINITION. A sequence a_0, a_1, a_2, ... is a *Cauchy sequence* if for any ε there is an $N \geq 0$ such that if $m > k \geq N$, then $|a_m - a_k| < \varepsilon$.

Note that a series satisfies the Cauchy Convergence Criterion exactly when its partial sums form a Cauchy sequence.

1. Show that a sequence has a limit if and only if it is a Cauchy sequence. *[10]*

Math Christmas Carols?

First, to the tune of *Santa Claus Is Coming To Town*:

> Oh, better take care completing the square;
> You'd better not try dividing by y;
> January exams are coming to town.
> We're making a list, don't shake in your boots;
> Just watch out for extraneous roots.
> January exams are coming to town.
> You know you'll have quadratics
> And exponentials too,
> You rationalize denomi-
> Nators like root two.
> So, you'd better be bright and calculate right,
> You'd better check roots for one that suits;
> January exams are coming to town.

Second, to the tune of *Rudolph The Red-Nosed Reindeer*:

> Zero, that funny cipher, has a shape that looks like "O,"
> And if you want to use it, there are things you need to know.
> Never divide by zero; if you do, you will be sad,
> Getting a crazy answer, making your report look bad.
> But treat zero as your friend – use him carefully –
> "Safe to multiply and add" – That's the rule for zero, lad!
> Zero, that screwball number wants to be a comrade true,
> But never divide by zero, or **you'll be getting zero, too!**

I don't know who wrote these lyrics; Melanie Goncalves gave me a copy of these and several other math take-offs on popular Christmas songs.