

## Mathematics 3790H – Analysis I: Introduction to analysis

TRENT UNIVERSITY, Fall 2009

### Assignment #1 – Is 'e crazy?!

Due on Thursday, 24 September, 2009.

In solving the following problem, you may assume without further ado that for any  $x > 0$  and  $n \geq 0$ ,

$$e^x = 1 + \frac{x}{1!} + \frac{x^2}{2!} + \frac{x^3}{3!} + \cdots + \frac{x^n}{n!} + R_n(x),$$

where

$$0 < R_n(x) < \frac{3^x x^{n+1}}{(n+1)!}.$$

(Not to worry, we'll show this is true later in the course.)

1. Show that  $e$  is irrational. [10]

*Hint:* Suppose  $e$  were rational. Try to derive a contradiction from this assumption by rewriting  $e$  using the expression above and then playing with it ...

#### There's a Delta For Every Epsilon

There's a delta for every epsilon,  
It's a fact that you can always count upon.  
There's a delta for every epsilon  
And now and again,  
There's also an  $N$ .

But one condition I must give:  
The epsilon must be positive  
A lonely life all the others live,  
In no theorem  
A delta for them.

How sad, how cruel, how tragic,  
How pitiful, and other adjectives  
That I might mention.  
The matter merits our attention.  
If an epsilon is a hero,  
Just because it is greater than zero,  
It must be mighty discouraging  
To lie to the left of the origin.

This rank discrimination is not for us,  
We must fight for an enlightened calculus,  
Where epsilons all, both minus and plus,  
Have deltas  
To call their own.

*Words and calypso music by Tom Lehrer.*