Mathematics 3770H – Complex Analysis
TRENT UNIVERSITY, Winter 2024

Assignment #9
Newton's Binomial Series
*Due on Friday, 22 March.**

As with all the assignments in this course, unless stated otherwise on the assignment, you are permitted to work together and look things up, so long as you acknowledge the sources you used and the people you worked with.

Suppose $r \in \mathbb{C}$ and $k \geq 1$ is an integer. Then the *binomial of* r *and* k is

$$\binom{r}{k} = \frac{r(r-1)(r-2)\cdots(r-k+1)}{k!}$$

and so we have $\binom{r}{1} = r$, $\binom{r}{2} = \frac{r(r-1)}{2}$, $\binom{r}{3} = \frac{r(r-1)(r-2)}{6}$, and so on. To make various formulas work nicely, we somewhat arbitrarily define $\binom{r}{0} = 1$. Note that when r is a non-negative integer, this coincides with the usual definition of binomial coefficients. It's a bit harder to justify reading $\binom{r}{k}$ as "r choose 0" when r is not a positive integer, though a lot of people do this anyway.

One of the main uses of this notation and terminology is to be able to conveniently state the following result.

NEWTON'S BINOMIAL THEOREM. Suppose a, $r \in \mathbb{C}$ with $a \neq 0$. Then

$$(a+z)^r = \sum_{n=0}^{\infty} \binom{r}{n} a^{r-n} z^n = a^r + ra^{r-1}z + \frac{r(r-1)}{2}a^{r-2}z^2 + \cdots .$$

It's not hard to check that when r is a non-negative integer, Newton's Binomial Theorem replicates the usual binomial expansion.

1. Suppose a, $r \in \mathbb{C}$ with $a \neq 0$. Determine the radius of convergence of Newton's binomial series, $\sum_{n=0}^{\infty} \binom{r}{n} a^{r-n} z^n$. *[5]*

2. Prove Newton's Binomial Theorem as best you can. *[5]*

* You should submit your solutions via Blackboard's Assignments module, preferably as a single pdf. If submission via Blackboard fails, please submit your work to your instructor by email or on paper.