Mathematics 3770H – Complex Analysis
Trent University, Winter 2024
Assignment #7
Due on Friday, 8 March.

As with all the assignments in this course, unless stated otherwise on the assignment, you are permitted to work together and look things up, so long as you acknowledge the sources you used and the people you worked with.

1. Suppose $f(z)$ is holomorphic on $\mathbb{C}$ and the real part of $f(z)$, $Re\,(f(z))$, has a maximum value of $e^1 3\pi^2$. Show that $f(z)$ is a constant function. *[6]*

Hint: Instead of working with $f(z)$ directly, consider $e^{f(z)}$.

2. Suppose $p(z)$ is a polynomial with complex corefficients of degree $n \geq 1$ in the complex variable z. Show that $p(a) = 0$ for $a \in \mathbb{C}$ if and only if $p(z) = (z - a)q(z)$ for some polynomial $q(z)$ of degree $n - 1$. *[4]*

Hint: One direction is trivial. The other direction requires some algebra, but not any calculus . . .

NOTE. Together with the Fundamental Theorem of Algebra, the result in **2** implies that every complex polynomial can be decomposed into linear factors.

Probable-Possible, my black hen,
She lays eggs in the Relative When.
She doesn't lay eggs in the Positive Now,
Because she's unable to Postulate How.

From *The Space Child's Mother Goose* by Frederick Winsor.