Mathematics 3770H – Complex Analysis [reading course]
Trent University, Fall 2022

Assignment #6 – Merrily We Go Around!
Due on Friday, 1 March.[*]

As with all the assignments in this course, unless stated otherwise on the assignment, you are permitted to work together and look things up, so long as you acknowledge the sources you used and the people you worked with.

Let C_r denote the circle of radius r in the complex plane with its centre at the origin, and oriented counterclockwise as a closed path.

1. Find all the points in $\mathbb{C}$ where $z^4 = -1$. *[2]*

2. Show that $\displaystyle\int_{C_2} \frac{1}{z^4+1}\,dz = 0$. *[9.5]*

Hint: This is pretty hard to compute directly. It is impossible to do it indirectly by shrinking C_2 homotopically to some point because the points you found in **1** are all inside C_2. However, you can use homotopy to show that $\displaystyle\int_{C_2} \frac{1}{z^4+1}\,dz = \int_{C_r} \frac{1}{z^4+1}\,dz$ for any $r > 2$, at which point an inequality we have for complex integrals may become useful.

Posted on `https://www.smbc-comics.com` on 2022-03-13.

[*] You should submit your solutions via Blackboard's Assignments module, preferably as a single pdf. If submission via Blackboard fails, please submit your work to your instructor by email or on paper.