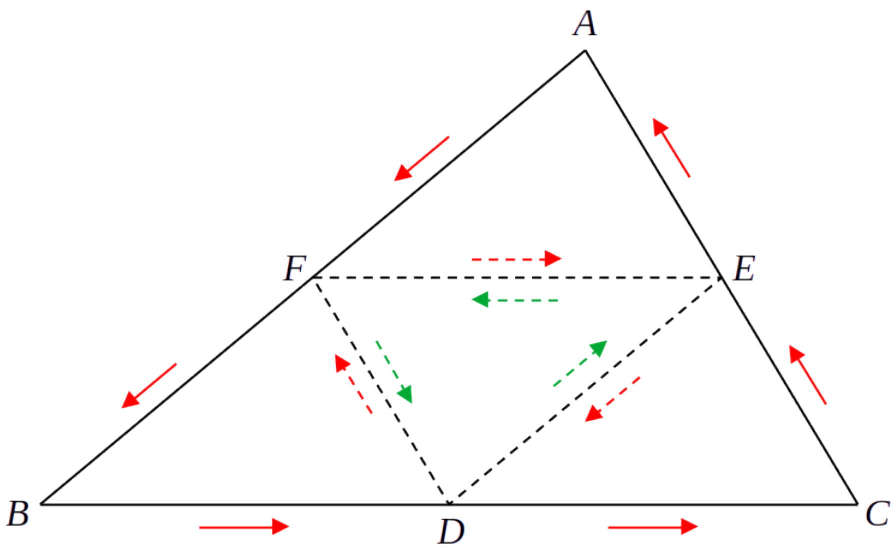
Mathematics 3770H – Complex Analysis
TRENT UNIVERSITY, Winter 2024

Assignment #5 – Cauchy's Triangle Theorem
*Due on Friday, 16 February.**

As with all the assignments in this course, unless stated otherwise on the assignment, you are permitted to work together and look things up, so long as you acknowledge the sources you used and the people you worked with.

The objective of this assignment is to prove a result due to Augustin-Louis Cauchy (1789-1857), one of the pioneers of complex analysis, along with several other fields.



LEMMA. Suppose the complex function $f(z)$ is holomorphic in the region $G \subseteq \mathbb{C}$ and $\triangle ABC$ is a triangle in G. Let F, D, and E be the midpoints of sides AB, BC, and AC, respectively of the triangle. Then

$$\int_{\triangle ABC} f(z)\,dz = \int_{\triangle AFE} f(z)\,dz + \int_{\triangle FBD} f(z)\,dz + \int_{\triangle DEF} f(z)\,dz + \int_{\triangle EDC} f(z)\,dz,$$

where in each integral, we run around the triangle counter-clockwise.

1. Prove the lemma! *[3]*

Hint: Follow the arrows!

THEOREM. Suppose the complex function $f(z)$ is holomorphic in the region $G \subseteq \mathbb{C}$ and $\triangle ABC$ is a triangle in G. Then $\displaystyle\int_{\triangle ABC} f(z)\,dz = 0$.

2. Prove the theorem. *[7]*

Hint: Suppose T is the triangle plus its interior. Let $t = \max\{\,|f(z)| \mid z \in T\,\}$. Note that each of the four subtriangles in the lemma has half the perimeter of the original triangle.

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* You should submit your solutions via Blackboard's Assignments module, preferably as a single pdf. If submission via Blackboard fails, please submit your work to your instructor by email or on paper.