

## Mathematics 3770H – Complex Analysis

TRENT UNIVERSITY, Winter 2024

### Assignment #4 – Projections

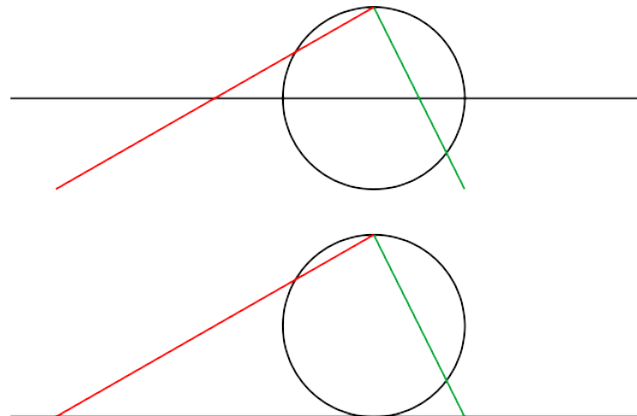
Due on Friday, 9 February.\*

As with all the assignments in this course, unless stated otherwise on the assignment, you are permitted to work together and look things up, so long as you acknowledge the sources you used and the people you worked with.

Please read (or at least skim :- ) §3.3 in the textbook. Among other things, this section describes a way of identifying the Riemann sphere  $\widehat{\mathbb{C}} = \mathbb{C} \cup \{\infty\}$ , which is discussed in §3.2, with the unit sphere centred at the origin in  $\mathbb{R}^3$  (i.e.  $x^2 + y^2 + z^2 = 1$ ) via stereographic projection, with the  $xy$ -plane in  $\mathbb{R}^3$  being identified with the complex plane via  $(x, y, 0) \longleftrightarrow x + iy$ . This projection works by taking the point  $(x, y, z)$  on the unit sphere to the point where the line of  $\mathbb{R}^3$  joining the north pole  $(1, 0, 0)$  to  $(x, y, z)$  intersects the  $xy$ -plane. The north pole itself is identified with the point at infinity, i.e.  $\infty$ , of the Riemann sphere.

A slightly different way of doing stereographic projection is to do the projection in the same way, but identify the complex plane with the plane  $z = -1$  in  $\mathbb{R}^3$  via  $(x, y, -1) \longleftrightarrow x + iy$  and take the point  $(x, y, z)$  on the unit sphere to the point where the line of  $\mathbb{R}^3$  joining the north pole  $(1, 0, 0)$  to  $(x, y, z)$  intersects the plane  $z = -1$ . The north pole itself is, once again, identified with the point at infinity, i.e.  $\infty$ , of the Riemann sphere.

Cross-sectional views of how these projections work are pictured in the diagram below.



Let's call the projection that takes points on the unit sphere to the Riemann sphere in the first version of stereographic projection given above  $\phi$  and the projection that takes points on the unit sphere to the Riemann sphere via the second version of stereographic projection given above by  $\psi$ .

1. What exactly is the mapping of the Riemann sphere to itself given by  $\psi \circ \phi^{-1}$ ? Is it 1-1 and onto? Is it differentiable? Is it a Möbius transformation? [10]

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\* You should submit your solutions via Blackboard's Assignments module, preferably as a single pdf. If submission via Blackboard fails, please submit your work to your instructor by email or on paper.