Real Differentiability?
Due on Friday, 26 January.
Due* just before midnight on Friday, 19 January.

As with all the assignments in this course, unless stated otherwise on the assignment, you are permitted to work together and look things up, so long as you acknowledge the sources you used and the people you worked with.

1. Suppose $f : \mathbb{C} \to \mathbb{C}$ is differentiable on all of $\mathbb{C}$ and $f(z) \in \mathbb{R}$ for every $z \in \mathbb{C}$, *i.e.* $f(z)$ happens to output only real numbers. Show that $f(z)$ must be a constant function. *[10]*

Sing me a song of the hydrogen light
Three degrees Kelvin illumine the night
Three degrees Kelvin, the infrared sky
Colors too deep for the unaided eye

Sing me a song of the hydrogen band
Whispering low since the cosmos began
Whispering low as the white light shifts red
Wavefronts of hydrogen sweeping ahead

Sing me a song of the hydrogen wall
Vector me out to that light bounding all
Vector me out in that glory to dwell
End of the universe, cosmic eggshell.

By John M. Ford, from his novel *Princes of the Air*.

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*   You should submit your solutions via Blackboard's Assignments module, preferably as a single pdf. If submission via Blackboard fails, please submit your work to your instructor by email or on paper.