**Mathematics 3770H – Complex Analysis**

Trent University, Winter 2024

**Assignment #11**

**Newton's Binomial Series Revisited**

*Due on Friday, 5 April.*[*]

As with all the assignments in this course, unless stated otherwise on the assignment, you are permitted to work together and look things up, so long as you acknowledge the sources you used and the people you worked with.

Recall the following from Assignment #9:

Newton's Binomial Theorem. Suppose $a$, $r \in \mathbb{C}$ with $a \neq 0$. Then

$$(a+z)^r = \sum_{n=0}^{\infty} \binom{r}{n} a^{r-n} z^n = a^r + r a^{r-1} z + \frac{r(r-1)}{2} a^{r-2} z^2 + \cdots .$$

**1.** Suppose we were to interchange the roles of $a$ and $z$ in Newton's Binomial Theorem and expand $(a+z)^r = (z+a)^r$ accordingly:

$$\begin{aligned}(a+z)^r = (z+a)^r &= \sum_{n=0}^{\infty} \binom{r}{n} a^n z^{r-n} = z^r \sum_{n=0}^{\infty} \binom{r}{n} a^n z^{-n} \\ &= z^r \left(1 + r a z^{-1} + \frac{r(r-1)}{2} a^2 z^{-2} + \cdots\right)\end{aligned}$$

If we continue thinking of $z$ as a variable and $a$ and $r$ as constants, what is the radius of convergence of this Laurent series? *[10]*

[*] You should submit your solutions via Blackboard's Assignments module, preferably as a single pdf. If submission via Blackboard fails, please submit your work to your instructor by email or on paper.