Mathematics 3770H – Complex Analysis
Trent University, Winter 2022

Assignment #6 – Around we go on in expanding circles!
Due on Friday, 4 March.†
(May be submitted on paper or via Blackboard.)*

As with all the assignments in this course, unless stated otherwise on the assignment, you are permitted to work together and look things up, so long as you acknowledge the sources you used and the people you worked with.

Let C_r denote the circle of radius r in $\mathbb{C}$ centred at the origin, oriented positively (*i.e.* counterclockwise).

1. Find all the points in $\mathbb{C}$ where $z^3 - 8 = 0$. *[1]*

2. Show that $\displaystyle\int_{C_3} \frac{1}{z^3-8}\,dz = 0$. *[9]*

Hint: This is pretty hard to compute directly. It is impossible to do it indirectly by shrinking C_3 homotopically to some point because the point(s) you found in **1** are all inside C_3. However, you can use homotopy to show that $\displaystyle\int_{C_3} \frac{1}{z^3-8}\,dz = \int_{C_r} \frac{1}{z^3-8}\,dz$ for any $r > 2$, at which point an inequality we have for complex integrals may become useful.

*A parody written after "studying algebraic topology for just a *little* too long":*

I think that I shall never see
A space as lovely as the tree.
A tree, where cycles don't occur,
A path-connected graph, for sure.
To single points it may retract–
To *any* of its points, in fact.
Each graph must have a group that's free,
But nothing's simpler than a tree.

By Stephanie Wukovitz. Posted to the newsgroup `sci.math` on 1992-05-23.

† March Fo[u]rth is the only day of the year that doubles as a command! :-)

* All else failing, please email your solutions to the instructor at: `sbilaniuk@trentu.ca`